Theory of Erosion on Soil-Covered Slopes

Azad Muhammad Tawfiq

٠٩/٢١/٢٠١٣

ABSTRACT

Slope erosion is subject to a minimal law; that factor, whether the rate of transport or the rate of weathering, which is relatively the least efficient, controls the general course of denudation. On soil covered slopes the rate of transport is the dominant denudational control, soil creep being the responsible agency. A previously proposed statistical theory of soil creep is outlined so that certain imperfections and errors may be noted. Methods of testing the theory are suggested and discussed briefly. Finally, some general aspects of the development of soil covered slopes are reviewed from the standpoint of the statistical theory. Slope erosion is, characteristically, erosion without corrosion. In contrast to all other aspects of erosion, transportation processes occupy a secondary role; with exceptions the actual movement of material on slopes does not of itself lead to the production of further removable material. The exceptions, significantly, occur at extreme positions, where the acceleration is such that falling rocks strike off splinters; where transportation is catastrophic, as in landslides; or where increasing amounts of water herald a transformation to fluvial erosion, as in sheet floods. On the majority of slopes transportation processes can only influence weathering processes in a negative manner; they are unable to increase the production of removable material though they may, by their inefficiency, bring about a decrease. Slope erosion is by far the least efficient means of landscape reduction; if it were not so, diversification of the terrain would more closely reflect structure and endogenetic activity. The relation between the rate at which the parent rock is reduced by weathering and the rate at which the products of weathering are removed provides a means of describing and classifying the properties and behavior of slopes subject to erosion. The two modes of erosion interact over a continuous range; at one extreme the influence of weathering is predominant, and the surface of the landscape is covered with a thick ١ Manuscript received March ٣, ١٩٦٤. layer of weathered rock material; at the other extreme the agents of transportation are capable of stripping a slope bare of all removable rock fragments. Within the transition zone the subordinate mode qualifies the performance of the major by reducing it below the potential set by the given conditions. In neither case can the action of one mode accelerate the activity of the other above the potential as set, and to reach this level will require the elimination of the other mode as an effective agent of erosion. In slope erosion we are concerned with a minimal process of a type common to many fields of study, pedology (Liebig's law of the minimum), chemical kinetics, and especially economic theory. In fact, by regarding the agents of transportation as the creators of a demand for weathered material and weathering as a means of supplying transportable material, denudation of the landscape can be reduced to a problem of supply and demand. The rate of weathering depends upon a host of conditions so that its range of variation is considerable, while the variation itself is liable to rapid, if not discontinuous, change both spatially and temporally. This is in marked contrast to the variables involved in the rate of material transport, especially where, as on soil covered slopes, they may be regarded as continuous and of limited variation. If denudation were merely a matter of transportation, a great deal of the diversity would be lost to the landscape.

On the other hand, the existence of regular surfaced landforms implies that transporting processes are able to assume a predominant role. On surfaces of low relief and adequate rainfall, soil creep is the responsible transport agent and supplies the dominant mode of denudation, with the result that structure is masked by regularity of surface form. It is only when dissection increases slope declivity, so increasing the effective demand of the transporting agents up to the level of supply that variations in rock resistance are able to make themselves felt. A structurally determined landscape is one in which the transporting agents on the slopes are performing below capacity. Soil creep is by no means the only agent of material transport, even on gently graded soil covered slopes. Without considering fluvial, glacial, or Aeolian activity, landslides and rock slides, rock falls and mud flows and all forms of mass movement provide in detail for a more efficient form of transportation. But whereas these other processes are isolated and restricted spatially and in most cases are ephemeral, soil creep takes place whenever and wherever there is a continuous cover of transportable material. Soil creep is characteristic of denudation in humid, temperate regions. Macroscopically it is a continuous process, and the consequent regularity of landscape form readily lends itself to idealization of a cyclic nature. In this article the class of soil covered slopes is defined as coextensive with the class of slopes subject to soil creep, irrespective of whether the surface cover measures up to the requirements of the farmer, engineer, or pedologist. Soil creep as a transport agent is confined to the upper layers of the surface cover. If the thickness of the cover exceeds the depth of macroscopic flow, soil creep will proceed unimpaired. It will be the controlling factor in slope erosion and will set the character of the landscape. The process can be idealized mathematically and a model constructed to cover all aspects and cases within the ideal humid cycle and, by extension, all soil covered slopes. Where the rate of removal of material from the surface is such that, despite a possible acceleration in the rate of weathering brought about by the removal the thickness of the soil layer is steadily diminished, there will come a time when the zone of macroscopic creep impinges on the horizon of imperfectly weathered rock. The activity and influence of soil creep then becomes subject to qualification by the rate of weathering. If the compensatory effect due to the decreased mobility of the partly weathered rock fragments is overridden by increased declivity, a more efficient mode of transportation, or by any other factor, the slope will eventually reach the stage at which material is removed almost as fast as it is rendered removable. Demand outstrips the supply. It matters little how efficient the agents of transportation may be; without a sufficient supply of material they necessarily operate below capacity. In general, the more efficient a transport agent, short of corrosion, the less likely that it will have any influence on the course of slope erosion. On "transport slopes" the rate of weathering assumes dominance as a denudational control. It also increases to the maximal value for the conditions as the protective mantle of weathered rock is reduced to a veneer of fragments in relatively rapid transit, which, owing to the insufficiency of supply, fails to provide a continuous cover. Denudation controlled by the rate of weathering is characteristic of semiarid erosion and all erosion on steep or elevated slopes, in fact on all slopes other than soil covered and those exempt from corrosion. It is not one system of erosion but many, none of which lend themselves as easily to mathematical idealization as does the erosion of soil covered slopes. Differences in rock resistance are sought out and accentuated in the surface form; the influence of geological strata becomes paramount. In broad outline the landscape will be structurally determined; in detail, by chance differences in lithology. Weathering unimpeded by a protective layer, or more generally, impeded to the same degree over the whole of a slope, will act uniformly normal to the surface so that, apart from end effects and subject to the vagaries of weathering in detail, the slope will instantaneously retreat parallel. Persistent parallel retreat will only occur where overall uniformity of weathering can be maintained. End effects disturb this uniformity by interfering with the flow of material and, through the medium of the mobile cover, the disturbance is propagated throughout the whole of the slope profile. Apart from special cases in which the behavior of the boundary controls imposes a nice adjustment between the rates of material transport and weathering, slopes with a curved (sigmoid) profile are all end effect; and any parallel retreat is coincidental and ephemeral. On slopes with a linear profile steep enough for the rate of material transport to strip the surface bare and so remove the agency of propagation, and with a completely absorptive lower boundary control enabling these conditions to be maintained, end effects are kept to a minimum and parallel retreat is persistent. The erosional system associated with the name of W. Penck is a special extreme case of erosion on non-soilcovered slopes. On steep bare rock slopes the rate of weathering is at an absolute maximum; but in relation to the potential rate of transport, which for free rock fall is effectively infinite, it is at a minimum. On the other hand, on gentle slopes where denudation is controlled by the rate of transport, the rate of weathering, although at an absolute minimum, is, when compared to the low rates of transport, at a relative maximum, so that, as a general principle of slope erosion, the course of denudation is determined by that factor, whether material transport or weathering, which is relatively least efficient. This idea can be represented graphically, the relative rates of transport and weathering supplying the two sets of values. Over the rectangular domain so defined, every point represents a possible erosional system or relationship between the two modes, although some will be physically most improbable. The Davisian scheme of erosion will be represented by points near to the origin; the Penckian scheme at the opposite pole, where weathering is at a maximum (say, unity) and the potential rate of transport infinite. This diagrammatic representation of slope erosion will not be discussed further, since it is introduced to illustrate the concept of a continuous spectrum of erosion systems and the polarity of the Davisian and Penckian schemes.

THEORY OF SOIL CREEP

From the fundamental hypothesis that soil creep is the macroscopic expression of minute random displacements executed by individual particles it can be shown that an aggregate of soil particles is subject to transport phenomena of the diffusion type (Culling, ١٩٦٣). Soil particles are regarded as subject to various processes: thermal expansion, capillarity, micro faunal activity, that tends to displace particles individually. As soil particles are in contact the distribution of displacements is determined by the distribution of pore space surrounding the particle. This is assumed to be randomly distributed about the center of the particle, from particle to particle, so that, for the aggregate, displacements are distributed randomly as to direction. The particles are in loose contact; the length of each displacement will be small; a displacement length equal to the average particle diameter will be rare. If the pore space is distributed randomly about each particle, there can be no autocorrelation between successive displacements as to length or direction. Each displacement is independent of all previous displacements, with the exception of the immediate reversal to the previous position. The vacancy left by a displaced particle tends to favor an immediate return displacement; the net effect on the particle is as if it had remained stationary. This "vibratory" behavior of the particle tends to increase the chances of displacement in neighboring particles and therefore increases the microscopic mobility of the aggregate; otherwise it may be neglected so that we may concentrate on the successful displacements. These make up only a small proportion of the number of times the forces tending to displace the particle are operative, which are themselves intermittent in operation; so for the greater part of their existence soil particles are stationary. In my previous paper (١٩٦٣) it was assumed that the distribution of displacements tends with increasing number to the normal distribution about the center of a particle. Also it was implied, if not actually explicit, in the argument that the displacement lengths are equal. Both these assumptions are redundant, the second being totally unrealistic for the movement of particles in soil aggregates. From the fundamental hypothesis that particle displacements are governed by the distribution of adjacent pore space and that this is distributed randomly about each particle and from particle to particle, it follows that each series of displacements made by a particle forms a sequence of independent random variables; and under fairly general conditions the central limit theorem in the theory of probability states that the sum of a large number of independent random variables approximates a normal distribution. The possible displacements open to a particle at any one instant form a statistical population conditioned by the physical configuration of the adjacent pore space and that is thereby rendered finite and discontinuous. From this three-dimensional population we may derive a one-dimensional population, finite and discontinuous, by projection onto the x-axis. The actual displacement of the particle, or its projection, is as if selected at random from the appropriate population. The probability that it will bear an assigned value is given by the appropriate frequency function. Once the displacement is effected, the particle is presented with a fresh configuration of adjacent pore space and therefore a fresh population of possible displacements from which one is selected at random at the next displacement, and so on. We thus define a sequence of random variables X١, X٢,..., for which, in each case, the mean value is zero or can be made so by the addition of a constant and the standard deviation is clearly finite from physical considerations. If X, is the random variable, then o, is the standard deviation and F,(x) is the distribution function, this latter giving the probability that a member selected at random from the given population will have a value equal to, or less than, a specified value. Denoting the sum of the standard deviations by S,٢, then as n increases, S)٧ ٢ ... and ٠. Sn While the total standard deviation increases beyond all bounds as the number of components increases, the contribution from any individual component becomes a decreasingly small fraction of the total. No one component is predominant or "large." These conditions are not quite equivalent to limS ٢ lf x٢dF,(x) = ٠ (e>٠), the LindebergCramer condition; but if the absolute moments /kv of order k > ٢ are less than a positive constant M, which is, at least, a plausible assumption for natural soil aggregates, then the condition is satisfied completely and it follows that the distribution function of the sum of a large number of components tends to the normal distribution, S e/٢dt (Cramer, ١٩٦٢, p. ٥٨٦١). The possible displacements open to a particle at any one instant provide a set of directions that form a subset of the totality of all directions in three dimensions. At each displacement a fresh subset is presented as if selected at random, there being no restriction on the selection with the exception of particles on the surface, or, possibly, within a zone of partly weathered, bedded, or jointed rock. Now the result of the central limit theorem can be generalized to cover the three-dimensional case, requiring that every component random variable has a probability function with vanishing first moments and finite second moments and satisfies a condition analogous to the LindebergCramer condition quoted above (Cramer, ١٩٦٢, p. ١١٤). Apart from the exceptions noted, which will be discussed later, these conditions will be satisfied by all soil aggregates. The weaker result concerning equidistant displacements is seen to be a special case, as is vibratory behavior. Here the distribution function is a step function, or a stepwise function if vibration takes place in more than one direction. By pairing the step functions the first moment vanishes immediately; for the stepwise functions this can be achieved by the simple addition of a constant. Clearly the standard deviation will be finite. From physical considerations we can assume the higher absolute moments to be bounded; so, provided the particle does succeed in getting displaced every so often and at each displacement the vibration is distributed randomly as to direction and length, as will clearly be the case, the distribution function of a large number of vibrations will approximate the normal. From the reproductive or group property of normal distributions the distribution function of the sum of a sequence of normal distributions is itself of normal form. Consequently, the sum total of all displacements of all particles within a soil aggregate over a sufficiently long time interval will have a normal distribution. This sort of statement about the probable behavior of a very large number of soil particles is the most general statement that the theory of soil creep can make. It cannot advise as to the precise behavior of individual particles; nor is it necessary or desirable that it should. In its purely mathematical aspects the theory of soil creep is a branch of probability theory and as such is ultimately concerned with the theory of additive set functions. But the theory of soil creep is not a purely mathematical theory; it is essentially a theory about the real world. Mathematics is unable to come to grips with reality, whereas the ٢٣٤ theory of soil creep claims to predict the probable behavior of natural soil aggregates. Whether any assertion about the landscape is true or false can only be decided by a model based on, and tested by, observation. The theory of soil creep bears several close resemblances to the theory of Brownian movement. For a suspended particle below a certain size, the number of impacts experienced during a small time interval fluctuates to an extent that renders the resultant of sufficient magnitude to displace the particle. The continually changing pattern of impacts arising from the molecular chaos within a fluid medium induces a sequence of randomly orientated movements of variable but minute extent. The erratic dance of fine particles suspended in a fluid attracted the notice of the botanist R. Brown in ١٨٢٧. Subsequently a variety of explanations was offered, summarized by Furth (١٩٢٦, p. ٨٦), but it was not until ١٩٠٥ that Einstein supplied a satisfactory explanation (Einstein, ١٩٢٦). In soil aggregates the particles are in contact, and the reaction of neighboring particles acts as a constraint upon the freedom of movement; but, as the permitted avenues of displacement change direction in a random manner from particle to particle and from time to time, the behavior of soil particles is similar to that of particles in Brownian motion. As we have seen, even if each particle has only one avenue of displacement open at any one instant and the interval during which it is open occurs intermittently throughout a period when no avenues of displacement are available, the behavior will still tend to give a normal distribution of displacements as the number, or alternatively the time interval, increases beyond all bounds. The number of "vibrations" and abortive displacements is high compared with the successful displacements. The relative freedom of the particle determines the magnitude of the diffusion coefficient, but it has no effect upon the form of the motion. The absence of a free mean path, in the stricter sense, in soil aggregates means that the value of the diffusion coefficient is very small indeed. If compressed air is blown through a dry granular aggregate, the resemblance to Brownian motion is increased as the relative magnitudes become more comparable. As the particles separate under the impress of air, their freedom increases and they are able to partake of the turbulent motion of the medium. Water is also able to separate soil particles and induce fluidization. In my previous paper (١٩٦٣) it was suggested that sufficient water and pressure to produce significant separation will lead to some form of plastic flow on slopes of sufficient gradient, there being no transition zone between random granular diffusion and directed plastic flow. Recent work by Dr. B. Kirby at Cambridge, England (personal communication) suggests that this view is incorrect and that directed movement under the influence of gravity can take place without necessarily being of a plastic nature. If we project the motion of a soil particle on to the (horizontal) x-axis, we have an example of one-dimensional random walk. We already know the position xo at time t = ٠ of the particle, and we need to determine the probability f'P(xo x; t)dx that at time to the projection of the position of the particle will be found to lie between xl and x٢. From probability theory we must have P>٠, P(xo Ix; t)dx = I , and also limP(xol x; t) = (x x), t*O where ٦ (x) is the Dirac delta function with the properties S(x) =٠ , x O , f_ (x)dx= I. The first two conditions express the fact that the particle exists and is conserved, and the third that it can certainly be found at x = xo at time t = ٠. The solution originally given by Einstein (١٩٠٥) for a free particle in Brownian motion is P(x, t) = e(x)/٤Dt (١) which is the fundamental solution of the diffusivity equation &P a٢P = ax٢ (٢) P(x, t)dx gives the probability of finding a particle, initially at x = xo, within the region x, x + dx after a time t. Another important interpretation of P(x, t) is that, if a large number of particles are concentrated at x = xo at time t = ٠, after time t has elapsed the proportion of the original concentration to be found within the region x, x + dx is given by P(x, t)dx. Both interpretations have been used to substantiate the Einstein expression. The first interpretation, involving the observation at successive time intervals of one, or possibly a small number of particles and determining the "relative residence time" or the percentage of the total time for which a particle is to be found within the region x, x + dx, provides a "time summation," whereas the latter interpretation involves a "space summation" at an instant. Although both give the same result, conceptual difficulties lying at the foundations of statistical mechanics debar the automatic substitution of space and time summations. This exchange is permissible without further considerations only if we are dealing with a so-calledErgodic system (Furth, ١٩٢٦, p. ٩٩). Further, a time sequence concerning the relative residence time, such as the observation at successive intervals of the number of particles within a specified region, does not as such constitute a random sequence or "collective" due to probability aftereffects. That is, the number of particles at a time t+i within a specified region is not independent of the number at time tn, where the interval between the two observations is comparable with that between successive displacements. But from the viewpoint of R. von Mises (١٩٥٧, p. ١٩١) it is possible to convert the sequence into a collective, and the probability of the relative residence time being different from the combinatorial probability becomes negligible for large numbers of observations. From expression (١) the mean square displacement is Ax= fx٢ (x, t)dx J (٣) = ٢ Dt . This is of great importance in the theory of Brownian motion and possibly of soil creep, as it can be observed and measured. From investigations into osmotic pressure Einstein derived the formula RT I D (٤) Nf' where R is the universal gas constant, T the absolute temperature, and N Avogadro's number, thus bringing out the thermo molecular nature of osmotic pressure. The friction term I/f is determined from Stokes's law if the mean free path is small (liquid medium) or from Doppler friction if it is large (gases). In soil creep the counterpart to the term RT/N is sought in the expression of the various forces tending to displace the particle and, in effect, the amount of energy expended in displacement divided by the number of particles. It may be possible to find a formula for this term, but there can be no simple relationship as for ideal monatomic gases. The friction of the medium is supplied by the reaction of neighboring particles and is therefore inversely proportional to the voidage but in no simple manner. Many factors besides the actual volume of pore space relate to the ease of movement of particles within the soil aggregate. Within the soil, assumed isotropic to particle diffusion, the concentration, C, of particles is governed by the diffusivity equation ٣C =div(D grad C); (٥) SJ and the relation between the rate of material flow and the concentration gradient is given by J= D grad C, (٦) where J is the diffusion vector, C the concentration, and D the diffusion coefficient. If diffusion takes place within a medium possessing a mass flow, the equations describing the diffusion are superimposed upon the equations of motion for the medium as a whole. Assuming that the latter set of equations is known and that they have been integrated so that we have knowledge of the velocity field, we may introduce the field effect into the model as an external force with the result that a mass transport term is added to the diffusivity equation ٣C = DV٢C v grad C, (٧) where D has been taken as a constant. Anisotropic diffusion is a possibility in certain types of surface flow, particularly with shale or slate fragments. In this case the nature of the diffusion coefficient becomes complicated, and equation (٧) needs to be generalized to ٩C V VC aC .i , ,(٨) at xaD ٨xx where Dij is a tensor which in the most general case may depend upon vi as well as xi. This very general equation has been used by Scheidegger in the study of flow through porous media (١٩٦٠a, ١٩٦١c), and it may eventually prove necessary to do likewise in the study of surface movement of material on slopes if an accurate representation at the soilmechanics level is required. It is far too general for any easy application to specific geomorphic problems. In the soil cover on hillside slopes equilibrium conditions are soon imposed in the vertical plane. After an initial period of settling the distribution of displacements assumes the normal symmetrical pattern. Allowing for compaction, the concentration of particles in a vertical plane can be converted into a length, the soil thickness Z. The vertical component may be taken as a dependent variable and equation (٥) replaced by (D + f(٩) at d x٢ ' which must be further amended; for unless the soil thickness Z can be measured from a standard elevation throughout, which will rarely be the case, it must be replaced by z, the elevation. Material flow is proportional to the concentration gradient, in this instance the surface gradient, and not the soil thickness, provided this is above a certain value. For soil creep in its simplest form then, the equation governing the surface elevation on hillside slopes and determining the denudation of soil covered landscapes is az D(٢ a ٢ z =D x +٢ . (١٠) at ax٢ ay٢i Before the simple theory of soil creep can be applied to the problems of denudation, various subsidiary hypotheses are required, the most important concerning the depth of the soil cover. In my previous article (١٩٦٣) it was assumed that soil creep was confined to the uppermost layers and that very few natural soil covers fail to exceed the required depth. This is now seen to be erroneous and is due in part to insufficient care in distinguishing between surface flow and soil creep within the soil aggregate. From the nature of particle displacements and their dependence upon the distribution of pore space, macroscopic flow will take place from any region of greater density toward a neighboring region less dense. The soil aggregate is subject to a density layering parallel to the surface; consequently flow takes place in the direction of any negative surface gradient, tending to replace the existing density layering with a horizontal stratification. Consider the diffusion of a suspension within a container in which, initially, the concentration gradient falls from left to right, there being no gradient in the vertical plane or in the width of the container. Macroscopic flow will take place from right to left at a rate depending upon the concentration gradient; microscopically it is equally liable to take place at any point along a given vertical meridian. If we insure that enough particles are involved almost equal amounts of particles pass through equal intervals along any vertical meridian. If we now imagine the particles settling out at the base of the container so that they are just in contact, the concentration gradient will now be represented by the surface gradient of the accumulated particles. Diffusion is assumed to continue, but as the particles are no longer free it is dependent upon the presence and distribution of pore space within the sedimentary aggregate and in this respect is analogous to particle diffusion within soil aggregates. The diffusion coefficient is decreased, and markedly so, for not only is the mean free path reduced to zero, or almost so, but the number of displacements is now only a small fraction of the number of "vibrations." But the value of the diffusion coefficient is constant throughout the sediment; for the density of pore space is assumed to be uniform, with random fluctuations, throughout the entire accumulation, apart from the surface layer where the particles are still in some measure free. Macroscopic flow will still take place at a rate proportional to the concentration gradient (now the surface gradient of the sediment) but will be restricted entirely to the uppermost layers for it is only there that a concentration gradient occurs. The body of the sediment is of uniform concentration on the macro scale. Now if we consider the weight of the particles, the ensuing compaction will result in the proportion of pore space falling in general as the depth increases to a value corresponding to a close packed arrangement. Beyond this, it remains approximately constant until the weight of the superincumbent particles produces compression and distortion of the particles themselves. If this takes place on a wide scale so that individual distortions and readjustments augment one another, plastic yielding will take place if there is space available for the moving material to occupy. This can be provided for by the whole sediment mass flowing so that the upper sections occupy the space formerly occupied by the lower sections or by the production of a curved sole intersecting the surface and enabling material to override into the space above the former surface. Assuming that the portion of easily compressed pore space decreases linearly with distance from the surface, then, apart from slopes of zero gradient, there will be a slight difference in particle density between adjacent vertical columns at all levels down to the level of close packing. Macroscopic flow is still proportional to the surface gradient but is no longer confined to the uppermost layers. On the macro scale a small amount of flow takes place down to the layer of close packing. The value of the diffusion coefficient falls as the proportion of pore space falls from a maximum at the surface to zero at the horizon of close packing. Due to the greater mobility at the upper horizons most of the macroscopic flow takes place near the surface, and particles are redistributed vertically. Apart from the fact that compaction is not a linear function of weight but becomes increasingly more difficult, further factors in natural soil aggregates operate to oppose compaction in the upper layers. Many agents, and not the least the presence of root systems and the activity of earthworms, tend to aerate the upper layers of the soil and so increase particle mobility. Diffusion of particles enables "holes" to penetrate the soil, leading to a general "isothermal" layering of the easily compressible pore space. This ideally decreases exponentially with distance from the surface. The concentration gradient between adjacent columns will likewise decrease exponentially with depth; and macroscopic flow will dwindle to negligible proportions, the depth required depending upon the surface gradient. The overall effect will be to increase the mobility in the upper layers and extend the zone of highest mobility down from the surface and at the same time decrease the mobility in the lower layers, and therefore the probability of any macroscopic flow taking place there. The greater part of particle flow in detail takes place in the uppermost layers. The density layering is then readjusted by vertical movements acting in opposition to gravity. Although the diffusion coefficient varies in magnitude throughout the depth of the soil layer, it is the value near to the surface where it is at a maximum that is of importance in denudation. If the depth of the soil is adequate, we are justified in taking the diffusion coefficient as a constant. Surface flow differs from soil creep in that the one concerns the behavior of free or semi free particles; the other of aggregates whose mean free path is zero. The contrast between the two modes of transport is most apparent where vegetation is absent. The surface layer is unencumbered and unprotected against any external agent that may tend to initiate or sustain particle movement. Surface flow is so favored that even where the soil structure is such that macroscopic flow extends to a significant depth, the actual particle movement is predominantly at or near to the surface and redistribution vertically accounts for most macroscopic flow at depth. Vegetation reduces the freedom of surface particles; and a continuous cover heath, grass, or forest with undergrowth is so effective that particle mobility falls to a value appropriate to the upper layers of soil. Surface flow is, in effect, reduced to soil creep; and a cover of vegetation is often the significant factor in determining whether a slope is to be regarded as behaving as a soil covered slope. Surface flow on bare or sparsely vegetated slopes differs from soil creep in its detailed mechanism as well as its magnitude, although in many cases it can still be described by a diffusivity type equation. SURFACE MOVEMENT The acceleration of particles by external forces on gentle slopes can take place only in conjunction with prior displacement by forces operating within the soil aggregate itself. The external forces effect a secondary displacement most easily when the primary displacements are frequent and with increasing difficulty as the mobility of the surface particles diminish. Secondary displacements by external forces are restricted to the surface layer, for displacement within the bulk of the surface cover requires a separation of the particles that, if found in a natural soil, would lead to a failure of the internal resistance and some form of plastic flow. Creeping flow in fluid media can be regarded as diffusion in a preferred direction. From a micro model designed with an asymmetric frequency distribution of particle displacements it can be shown (Culling, ١٩٦٣, p. ١٤١) that the diffusion vector needs to be generalized by the addition of a mass transport term, J = D (az/Ox) fZ, leading to the equation az ٢z٢ ٠ = D (Z), (١١) a t ax٢ ax where ٣ = F,, a "force" directed parallel to the x-axis, and Z the thickness of the layer involved in the mass transport. The external force produces a mass velocity upon which the diffusion is superimposed, though physically the diffusion is prior. On soil covered slopes the most important external force is directed downslope. On gentle slopes, unless a particle is first dislodged by frost heaving, thermal, or moisture expansion, raindrop impact, removal of support, or any other agency, the acceleration of gravity will be unable to overcome the resistance offered; otherwise particles would be removed from the surface as soon as weathering renders them removable, as indeed is the case on very steep slopes. The particle travels downslope until the resistance of the surface again brings it to a standstill. The frequency of dislodgement will be small, as will be the time interval of travel compared with the stationary period so that successive displacements rarely overlap. If the frequency of dislodgement is raised (this can be achieved experimentally by shaking or blowing compressed air through a granular aggregate), the average time interval between dislodgements becomes less than the average interval of travel so that particles rarely come to rest and surface movement becomes a continuous flow. The relatively rapid flow of sand in a tapped box is an extreme example but is nevertheless subject to the same principles as the slow flow of rock material over the slope surface; namely, the secondary directed redistribution of particles by the external force operates after, and only after, the primary random displacements caused by processes internal and peculiar to the system. The acceleration downslope for a completely free particle is replaced by a drift velocity. For displacements of small extent, of the same order as the average particle diameter, we may take the average length of displacement due to the external force as proportional to the strength of the force; and so Fx is assumed to vary with the sine of the slope angle ٦. To derive the component in the x direction a further resolution gives F,=Fsin ٦cos =Fdydx. dsds On slopes of gentle gradient (less than ٢٠°) we may replace the sine of the slope angle by the tangent and the cosine by unity; thus F dy dx' F being directed as the negative gradient. Substituting in equation (١١) gives z a٢z a= (D + FZ) >O tO X٢ ٣٢zX For all natural soil covered slopes of gentle gradient, with vegetal cover, it was asserted in my earlier paper that FZ is the minor component in the composition of K. This is not necessarily the case, however. On slopes subject to the artificial interference of farming or merely of forest clearing, the mass transport term may become significant, if not predominant. Returning to the example afforded by a sand box which bears many resemblances to surface movement on slopes, (١٢)the relative magnitude of the diffusion term is decreased if the primary motion of the particles is biased vertically, as when the box is tapped on the base. The random element is supplied by the projection of this predominantly vertical movement on the horizontal plane. The mass transport term is important because the external force comes into operation at each particle trajectory and is roughly proportional to the intensity of the agitation. A similar situation in which the primary motion is in a preferred direction, supplying the random element by projection and in which any external force acts perpendicularly, occurs in the movement of beach material. By increasing the relative importance of the mass transport term the application of equation (١١) can be extended to a far wider range of transport phenomena than that found on soil covered slopes. Surface movement on uncovered slopes lies beyond the scope of this article and will be discussed only insofar as it throws light on the position of soil creep within the entire range of transportation processes. In all cases where an external agency operates upon an aggregate of fine materials to produce a general drift in a preferred direction, three components can be distinguished in the motion of the individual particles: the initial dislodgement, the drift trajectory, and the terminal process that brings each individual flight to a close. Multiflight displacements are a distinct possibility. Slope erosion provides a set of special cases. The three components are seen to advantage on slopes with a thin cover of rock material but with no continuous vegetation. Initial displacement may be produced by the same agents operating within the soil cover to produce creep, or they may be surface effects peculiar to bare slopes, such as raindrop impact. Finally, the motion of a transporting medium itself, or the impact of other particles in motion may be sufficient to dislodge a particle. The immediate neighboring positions are less stable for the majority of particles so that cessation of the displacing force is usually followed by the particle's return to its original position. For a small proportion the potential barriers are overcome and the initial dislodgement is followed by acceleration normally but not necessarily downslope. As for soil creep, successful displacements are assumed to be randomly directed, the frequency distribution for any large number of displacements, however arrived at, from one or from many particles, being symmetrical and of normal form when projected onto the horizontal plane through the initial position. Randomly directed displacements tend to space out a veneer of rock fragments. Density tends to uniformity with fluctuations for a closed region, to zero for an infinitely large region or one with absorbent boundaries, a steadystate distribution if material is supplied steadily to the system by weathering of the underlying rock. The spread will take place irrespective of gradient; the downslope drift will tend to weight the distribution toward a closed lower boundary and to bias the steady state distribution about a localized source of fresh material. But if the source is distributed uniformly, there will be no noticeable effect upon the overall density within a small sector of a larger region expanding into infinite space or within a region with absorbent boundaries. The continuous creation of material by weathering leads by expansion of the aggregate to a steady state universe of rock fragments. From the discontinuous veneer of rock waste there is a gradation to soil creep. As the thickness of the cover increases, some form of density layering eventually occurs; and in effect the twodimensional diffusion of the surface veneer is replaced by a threedimensional diffusion, the density layering producing a macroscopic flow proportional to the surface gradient. During downslope drift the longer traverses, apart from unimpeded fall from a cliff face, consist of a series of smaller displacements or flights distinguished from each other by changes in direction. These changes comprise the third component and are brought about by the reaction of the particles of the surface or possibly in sheet flow by the turbulence of the transporting medium. The path after collision depends upon the momentum of the colliding particles and their shapes and may follow any direction including the knock on or glide over and the recoil. Abstracting the downslope component ascribable to the external force, we are left with a set of displacements for which there is no autocorrelation between successive flights, no one particular direction is favored either in frequency or displacement length, and so from the central limit theorem the frequency distribution tends to the normal. The whole process resembles the diversion and displacement of tracers in fluid flow through porous media, termed "dispersion" by Scheidegger (١٩٦١d) to distinguish it from diffusion. Although the two processes are governed by the same type of equation, diffusion is due to unequally distributed bombardment of particles by outside agents, while dispersion results from the reaction of the particles of an aggregate through which flows a fluid with particles in suspension. Soil creep is a degenerate case in which the transport component reduces to zero and the diffusion and dispersion components coalesce. The diffusion brought about by various forces acting upon a soil particle cannot be distinguished from the dispersion due to the reaction of the adjacent particles. At the other extreme, the fall of a rock fragment from a cliff face is entirely a question of the transport component. In between there is a continuous range in the relative proportions of the random and directed elements. Rock fall on a steep surface in which the fragment bounces and glissades reveals a minor amount of random behavior. With decreasing gradient the magnitude of the external force declines and the dispersion component increases in relative importance. Whatever the rate of weathering a rise in the drift velocity will eventually raise the capacity of the transportation system above the rate of supply. The amount transported ceases to be a function of the transport efficiency and becomes dependent upon supply from upslope and by weathering. Equations governing the transport of material now have no direct relevance to the course of denudation. It is no longer permissible to transform equations for the concentration of particles to ones describing the behavior of the surface elevation. The equation in terms of the concentration of rock particles W is aW a٢W a = D (FW)+A(x), (١٣) at x٢ &x where A(x) represents the distribution of material added to the system by weathering per unit time and for which the boundary conditions are to be specified in terms of quantity or flux entering or leaving the system. Thus the transport of rock material on slopes is the outcome of a drift superimposed by downslope directed forces upon a background of noise. A complete evaluation requires the estimation of the diffusion and dispersion coefficients, the magnitude of the drift velocity, and, where applicable, the quantity of material supplied by weathering. The problem can then be resolved into one of boundaryvalue form for a twodimensional surface immersed in a threedimensional space. In general each coefficient and parameter is a function of the space and time variables. By varying the curvature of space and so complicating the geometry we may be able to simplify the physical equations; an idea that has had such brilliant applications may hold the key to a unified theory of slope erosion. Surface movement under the external action of gravity alone supplies the simplest case of equation (١٣), apart from soil creep which is all noise, for only the external force need be regarded as a variable; in certain cases this, too, may to a first approximation be taken as a constant or, as in the example already considered, a function of the surface gradient. A more complicated application arises where, as in rain wash, more than one type of agent is involved in the production of the initial displacements. Upon the continuous background noise responsible for soil creep the intermittent activity of rain wash superimposes a fresh set of displacements. Slope wash of the milder variety is to a first approximation a function of the overland flow, that is, f(x), where x is measured from the watershed. The diffusion coefficient is most frequently a decreasing function of the space variable, an effect brought about by the activity of rain wash in initiating particle displacements. These will possess a strong downslope component and as such are to be regarded as the first flights in the transport sequence, the contribution to the diffusion term being negligible. As the intensity of rain wash increases, to a first approximation downslope, the relative importance of the diffusion component declines until it is represented by the background noise alone. Not so the importance of the dispersion component, which will increase with the drift velocity. In sheet flow of the more violent kind the importance of the diffusion component is maintained by the turbulent nature of the flow. With sheet flow we enter a third class of erosional activity where the transporting agent itself is capable of producing its own supply of material by corrosion. Fluvial erosion in general falls within this class, and the transport of the bed load reveals the same three components. Turbulence supplies the greater part of the impetus for the initial displacement which is directed randomly throughout a hemisphere based on the river bed and centered on the particle. The particle then rolls or saltates downstream with the current; the dispersive element follows upon impact with other particles of the bed load and river bed. Suspended material partakes of the turbulent flow of the fluid. The general equation (٨) has already been proposed to describe the transport (Scheidegger, ١٩٦١c). The three classes of erosional activity are alike in that the transportation phenomena are describable by various special cases of the general diffusivity equation; they differ in the effect the transportation processes have upon denudation. If transport is weak and the rate of weathering strong, denudation is controlled by the transport process and is almost entirely a matter of diffusion, as in soil creep. Where transport is stronger compared with the rate of weathering, it is the latter that is the predominant denudational control; the external forces supply the necessary increase in the transporting capacity. Finally, where transport is strongest, corrosion intervenes to shape the landform as in all forms of fluvial erosion. With large-scale deposition of material we return to the class of transport systems with adequate supply of material so that the slope form is controlled by the transport process and is describable in terms of equation (١١). In the building of alluvial fans material is introduced to the system at the apex of the fan. It is subject to the normal background of random displacements and the downslope drift due to gravity, as on all other slopes. Periodically flood water washes the slopes; and ephemeral radial distributaries superimpose a set of particle displacements akin to sheet flow, although a main distributary may disturb the radial symmetry into an elliptical or more complicated form. Finally, suspended material may settle out throughout the area of the fan. Some forms of volcanic accumulation, when viewed from this standpoint, involve the solution of equation (١١) in a rather simple form for a radially symmetric region with a point source at the origin (crater) or a distribution f(r, t) representing the accumulation of lava and ash upon the surface. With the cessation of volcanic activity or the depletion of the water supply on an alluvial fan both cases grade into that of normal surface movement on slopes with radial symmetry. With the development of a soil structure and the growth of vegetation we finally return to soil creep and the particular cases discussed in Culling (١٩٦٣, p. ١٥٩). Throughout the whole range of surface movement the responsible transporting process includes a stochastic component. This arises quite naturally from the discontinuous and irregular detail of the earth's surface.

CORROBORATION

A theory is scientific insofar as it is potentially falsifiable; scientific advance is the record of failure to discredit a theory. The statistical theory of soil creep outlined above and in greater detail in the earlier paper is open to investigation and possible corroboration under three general headings: pedological, geomorphic, and statistical. The pedological is held to include all methods of direct observation and measurement of particle movement in the field and laboratory. A straightforward adaptation of the techniques used in the classical investigations into Brownian motion by Perrin and Svedberg would be to try to observe the diffusion of soil particles in trial plots under natural conditions. The experiment would be prolonged, to say the least, and the accuracy required a matter of high technical skill. The use of radioactive tracers, as in the investigations into solid diffusion by Hevesy, eases considerably the problems of measurement, as it is no longer necessary to section or otherwise interfere with the sample to determine the particle distribution. The measurement of soil creep in the field is only now beginning to command the attention of geomorphologists and is proving to be a matter of some difficulty. The problem is how to measure the macroscopic movement within a soil aggregate without in any way affecting the movement on the microscopic scale. A further approach would be to adapt the laboratory technique to natural examples in the field. The junction of differing rock strata should be reflected along the outcrop in the composition of the derived soil cover, and creep will tend to blur the junction and displace it downslope. Additionally the junction is liable to be followed by a vegetation change. A statistical analysis of species distributions should provide a rough guide to the extent of particle displacement since the last time the surface was stripped of its cover during the Pleistocene, without the necessity for chemical analysis. For all varieties of the direct or pedological method it is essential to include in the research program the case of zero gradients. Soil creep is a known phenomenon on inclined slopes; but the theory of soil creep claims that this creep is, at least in part, the macroscopic expression of randomly directed displacements. Therefore particle diffusion should take place even where no macroscopic flow takes place, that is, upon a level slope. An appropriate experiment would be to contrive a vertical junction within a soil sample under natural conditions, separating a section that includes an easily recognizable constituent distributed uniformly. If no diffusion should take place, then the theory of soil creep as presented has no scientific value, however reasonable, however elegant. The theory can in this very simple fashion be falsified by a single observation. From the viewpoint of Karl Popper the theory of soil creep is eminently scientific. "In so far as a scientific statement speaks about reality it must be falsifiable; and in so far as it is not falsifiable, it does not speak about reality" (Popper, ١٩٥٩, p. ٣١٤). There is one possible source of ambiguity. Given the random behavior of soil particles, the recurrence of the initial distribution, assuming no chemical change, is possible; but it is remote. For a finite aggregate, given sufficient time, any possible distribution, defined by the spatial coordinates of all constituent particles, must recur, not exactly, but to any assigned degree of accuracy and infinitely often. The time interval required for this quasiperiodic behavior depends upon the degree of accuracy required and whether the specified state is a "probable" or "improbable" state of the system. In the experiment quoted the specified state possesses a systematic distribution with a high degree of order and is therefore a highly improbable state. The time required for a recurrence to a degree of accuracy set by the limits of observational technique is so vast that a negative result to the experiment can be regarded as decisive, provided the time exceeds that sufficient for a large number of displacements. The amount of particle diffusion in the experiment is certain to be small. Sir John Russell reports that fertilizers do not move laterally to any appreciable extent in level fields. As this evidence is to some extent against the conditions presupposed by the theory of soil creep, the original wording is preferable. "Thus a grass field at Rothamstead, originally uniform in its herbage, was in ١٨٥٦ divided into plots each of which actually touches its neighbors'; certain manurial treatments have been given annually to each and continued without change ever since. Marked differences in herbage have resulted, but the edges bounding the plots are fairly sharp; there is no evidence of much lateral diffusion in the period of ninety years" (Russell, ١٩٥٠, p. ٤٤٦٤٤٧). However, because of soil structure and conditions, even from the activity of earthworms in the soil, it is unlikely that a soil aggregate would ever remain static. The question is really how much particle movement takes place; what is the magnitude of the diffusion coefficient; and is it sufficient to explain the landscape forms attributable to soil creep? Whereas pedological methods attempt to assess the micro diffusion of soil particles, the geomorphic and statistical are on the macro scale and concern the diffusivity equation irrespective of derivation and the ultimate nature of soil creep. Geomorphic methods involve the prediction of landscape form and a comparison with reality. The method is restricted to special cases, for only in very favorable circumstances is it possible to determine the initial and boundary conditions. In the previous paper a condition of steady state was deduced for the profile of uniformly undercut slopes, thus eliminating the difficulty with regard to the initial state. But the number of instances cannot be large, as the rate of undercutting is usually too great to allow soil creep to proceed unimpaired throughout the length of the slope. To date, despite considerable search and measurement, no completely unambiguous example has been found. For nonstationary cases the initial condition needs to be known, and this will almost certainly have to be the almost level surface, whether of erosional, depositional, or structural origin, or some other regular form (alluvial fans, volcanic cones), for it is doubtful if any less regular initial surface can ever be known with sufficient accuracy. Also some estimation has to be made of the movement of relevant local base levels, as these supply the boundary conditions. Even in the most favored and well-known localities our knowledge of past conditions falls far short of the accuracy needed for a significant assessment, and the behavior of stream profiles is not known in sufficient quantitative detail. Changes in the physical environment during the later Tertiary and Quaternary add their quota of indeterminacy. Finally, any problem suitable for a comparison with the real landscape will be sufficiently complex to rule out solution by analytical methods. Although the diffusivity equation is particularly suited to solution by numerical methods, the tracing of the development of a simple straightforward landform such as a small stream valley or an isolated semiconical hill in the detail required is extremely laborious and a major undertaking. It is not practicable to run off the solutions for several sets of initial and boundary conditions and compare results, unless perhaps one has access to a computer. The problem posed by the geomorphic approach can be summarized as the attempt to assess a quantitative boundary theory with only qualitative data regarding the boundary values and that of a rather vague and indeterminate nature. Nevertheless some attempt has been made to carry out an investigation along such lines, if only to see if the predictions of the theory give reasonable qualitative results and to gain some insight into the orders of magnitude involved for comparison with the results of the more direct pedological experiments. The application of statistical procedures to terrain analysis is usually of an investigatory nature, designed to discover relationships implicit in various aspects of landscape development. Inferences as to the probability of certain relations are made from observed frequencies. Statistical analysis can also be used in a corroborative role, and as such is particularly important in branches of science where controlled experiments are not possible. By using large numbers the result can be made independent of the initial conditions, so avoiding the difficulties attendant upon the geomorphic approach. The essence of the method is to determine whether the observed frequencies in the landscape form a probable state according to the model supplied by the theory of soil creep. But the whole approach needs very careful theoretical justification and is mentioned here only for the sake of completeness. The theory of soil creep supplies a growth model for individual landforms. In order to provide a growth model for landscapes in general that can be tested statistically, supplementary hypotheses are required. The spatial relationships imposed upon the landscape by the drainage system need to be inserted, but it is not at all certain that this can be done with sufficient precision or that morphometric considerations can supply the information. Then the time averages of the growth model are to be compared with the space averages from the landscape, a procedure difficult to justify. Ergodic problems of this nature were first explicitly introduced into geomorphology by Melton (١٩٥٨) in a remarkable paper on the geometric properties of mature drainage basins, though as there mentioned, they have been implicit ever since the classical period of geomorphic theory. Statistical methods may provide a means of avoiding the practical difficulties of the geomorphic approach but with considerable theoretical difficulty.

DENUDATION OF SOILCOVERED SLOPES

The products of rock weathering form a protective mantle, partly insulating the parent rock from thermal fluctuations and through which the agents of chemical weathering must diffuse. The protective effect operates to reduce the rate of weathering with distance from the surface, and any agency that disturbs the surface of weathered material will thereby affect the rate of weathering. On level surfaces with no macroscopic flow the thickness of the layer of weathered rock products continues to increase as does the protective effect until the rate of weathering of the parent rock is eventually reduced to negligible proportions. Weathering may continue, but at a greatly reduced rate that appears to be independent of the depth, to give very deep soils on undisturbed surfaces over long periods. If the surface possesses a gradient, soil creep will remove material downslope; unless this loss is made good by the advent of material from above, soil thickness will be reduced. Consequently, the rate of weathering will be accelerated until a fresh quasistationary state is attained where the advance of the weathering front will proceed at a rate set by the rate of removal of material from the surface, which in turn depends upon the rate of change of surface gradient. Movements of soil particles may take place at any level within a thick mantle, but the vast majority is distributed symmetrically and makes no contribution to macroscopic flow. Only in the upper layers will any interference with particle mobility be reflected in the rate of soil creep. Any agent tending to reduce the soil thickness and so cause the semi weathered horizon to rise toward the surface will produce no recognizable effect upon the course of erosion until the semi weathered horizon approaches the surface. The absence of soil structure, the larger and more angular fragments found in this horizon, and their ordered and interlocking arrangement inherited from the parent strata inhibit the mobility of the soil aggregate as a whole and may tend to make it anisotropic. Macroscopic creep remains proportional to the surface gradient but will be retarded by the lower values of the diffusion coefficient. Further net removal of material from the surface layers will bring unweathered rock to the surface soon to be followed by the stripping of all loose fragments. The diffusion coefficient is now effectively zero. To cover the continuous range over which soil creep cannot function at maximum efficiency a factor # is added to the right-hand side of equation (١١) to give c z ٢z a = D (#Z). (١٤) at ax٢ ax The factor t varies from unity on thick covers to zero on bare rock slopes, though soil creep will have ceased long before the lower value is reached. The magnitude of the factor will clearly depend upon the relationship between the rate of weathering and the rate of removal of material from the surface. The effect upon the diffusion coefficient is to render it a variable in both space and time. Determining the behavior of the factor requires a simplified idealization of the complex process of weathering. The problem may be approached by way of the two aspects of weathering already mentioned: the protective role of a mantle of weathered rock or the necessity for the agents of weathering to diffuse through the layers of weathered material. We may regard weathering as a species of surface corrosion and construct a model based on the behavior of oxidation films. If the rate of weathering is inversely proportional to the thickness of the weathered layer a simple model would give the socalled parabolic law. Thus if z is the thickness of the weathered layer and k a constant, then dz k = z; z= ٢kt (z = ٠ at t= ٠). This fits several tarnishing reactions, the value of k being dependent upon the diffusion coefficient of the reactant (Jost, ١٩٥٢, p. ٣٤٢). As it stands, this model is oversimplified in any application to weathering. No provision is made for a gradation between unweathered and completely weathered rock; the front is sharply defined. It is a macroscopic model, whereas a truer parallel would be molecular. The alternative approach based upon diffusion appears to be more flexible, on the correct scale and closer to the natural pattern, at least in humid temperate conditions; but it is unlikely that any one model will be adequate to cover the whole range of weathering phenomena. Ideally the investigation of weathering models should go hand in hand with field and laboratory studies, but the direct approach is hampered by the very slow rates involved. Falling back on a more geomorphological approach, a model of weathering can be used to predict slope profiles which may then be checked against the real landscape. A detailed description of the method is best left until we are in a position to make a field assessment. We may remark, however, that the problem is even more difficult than the geomorphic assessment of the theory of soil creep, for not only is it subject to the same ambiguities with regard to the physical conditions but the actual process of solution is more involved and laborious. Apart from possible but very special cases, the treatment of equation (١٤) even without the mass transport term must be numerical. At each stage of the calculations the value of the diffusion coefficient must be known at the outset, so that the presence of a time variable coefficient leads to difficulties requiring special treatment (Crank, ١٩٥٦, p. ١٩١). Progressive attenuation of soil thickness and consequent decline in the importance of soil creep are opposed by mechanisms arising from the relation between the rate of weathering and the removal of weathered material. The advance of the weathering front is accelerated by the removal of the upper soil layers, so tending to restore an equilibrium thickness. It will require a rate sufficient in itself to strip the rock bare or a continually accelerated rate of removal to defeat this tendency to a quasistationary state. The decreased value of the diffusion coefficient appropriate to the incompletely weathered horizons that appear toward the surface checks the rate of soil creep and therefore the rate of removal of material. The two processes tend to maintain a steady thickness of the soil at a value corresponding to the maximum value for the diffusion coefficient for the environmental conditions. The effectiveness of the two compensatory mechanisms depends upon the course of denudation and in particular upon the movements of the relevant local base levels. It is possible to show that after an interval, which will vary from slope to slope with the initial conditions, the rate of removal of material measured as depth of soil or loss of elevation steadily decreases for all points of the slope profile, with the exception of points at base level and possibly in the neighborhood of the watershed if this should migrate, and will continue to do so throughout the remainder of the cycle provided there is no external disturbance to the system. The rate of removal of material dz/dt, which is effectively the rate of denudation, is proportional to the rate of change of surface gradient d٢z/dx٢, so that to show a steady decrease in the rate of denudation we need only show a like behavior in the rate of change of gradient, or more loosely, the "curvature," for all points of the profile. The precise relationship already established in equation (١٢) is az ^٢z Kt =K Ox٢ Considering first the case of stable base level, the solution takes the form of a Fourier series with a negative exponential factor. Thus for the region ٠ < x < ١, taken from the watershed at x = ٠ to the base level at x = ١, we have, respectively, a reflective and an absorptive type of boundary condition. The unique solution is given by z =٢ cos (٢n + I)X Io ٢١ n٠ X eK(٢n+I)٢ t/٤٢ (١٥) f١o ,. (٢n + I)rx' , X f (x') cos ٢١ dx , where f(x') represents the initial profile at t = ٠ and is sufficiently arbitrary to impose no restrictions within the realm of soil covered slopes. The presence of the exponential factor renders the expression uniformly convergent for all x, ٠ < x < ١, for every finite t interval. As t increases, the higher harmonics are successively suppressed, the series degrading into the fundamental of steadily decreasing amplitude. The slope gradient, f'(x), is characterized by a sine series, and the rate of change of gradient or curvature, f"(x), by a further cosine series, both of opposite sign to the original profile curve, f(x), so that if, as is usual, the profile curve is convex to the sky, both f'(x) and f"(x) measured from watershed to base level are negative. Both series degenerate with time to the fundamental owing to the presence of the convergency factor. Consequently, after sufficient time has elapsed for all but the fundamental term to have been reduced to negligible proportions, the curvature will decrease in absolute value for all points of the profile in the region ٠ < x < ١, that is, apart from the point at base level, provided the system is isolated from extraneous influence. This holds true for the rate of removal of material from the surface, so that, if a slope has a continuous soil cover when it enters the declining denudation phase, it will retain it throughout the cycle. If soil creep is proceeding unimpaired at that instant, it will continue so; the influence of structure upon the landscape will never attain significance. A denudational system characterized by soil creep constitutes a self-perpetuating system. It will not spontaneously break down, alter its character, or otherwise interrupt the continuity of its slow progress to ultimate pen planation. It is the imposition of variable conditions from without, mainly through the drainage net, that postpones the inevitable end and disrupts the path thereto. The above treatment is unnecessarily restrictive. To await the reduction of the cosine series to the fundamental is to neglect the greater part of the cycle. The curvature f''(x) of the slope profile decreases steadily in absolute value for all points ٠ < x < ١ as soon as it becomes a strictly monotonic function, provided the slope profile(x) describes a convextothesky curve. Expression (١٥) can be put into the form , = + (٢n + I)x f(x) = a٢n+١COs ٢١ n=O X eK(٢n+I)lt/٤١٢ (١٦) where a٢n+١ is the odd Fourier coefficient. The coefficients are of order n^, but whatever value s may take, SnaeK(٢n+)٢W'/٤١''٢O as >x o. (١٧) n=٠ The ratio of the (n + I)th term to the nth is (n + I)٦ ^K(n+I)rt/٤١, (١٨) so that the cosine series comes to be dominated by the fundamental after a relatively short period, compared to the time required for the series to vanish. From the RiemannLebesgue theorem the Fourier coefficients a٢n+١ ٠ as n + o, so that s < ٠ and the reduction of series (١٦) is rapid. If f(x) is bounded and otherwise satisfies Dirichlet's conditions in the interval (٠, ١), invariably the case if f(x) represents a slope profile, then the Fourier coefficients an are less in absolute value than k/n, where k is some positive number independent of n. If f(x) satisfies the above conditions and in addition f'(x), the slope gradient, is bounded and otherwise satisfies Dirichlet's conditions, as will be the case if the slope profile does not include a vertical section, then the Fourier coefficients a, are less in absolute value than k/n٢ and the cosine series is absolutely convergent. In general if f(x) and its first (p I) differential coefficients are bounded, continuous, and otherwise satisfy Dirichlet's conditions and if the path differential coefficient is bounded, the Fourier coefficients a, are less in absolute value than k/np+'. If the slope profile does not include a vertical section or a sharp break of slope, the conditions are satisfied for p = ٢, at least, so that for all soil covered slopes the higher terms of the cosine series for(x) disappear in rather short order and for the vast majority in very short order indeed (Carslaw, ١٩٣٠, p. ٢٦٩٢٧١). From equations (١٧) and (١٨) and the analysis given above, the relation ٢٤٨ d^z , (٢n+ I)x dx٤ = Ca ٢n+l cos ٢l O٤ ٢١ X eK(٢n+I)٢'wt/٤١' > ٠ (٢٠) for all x, ٠ < x < ١, and where the coefficient a'٢n+ is of order n^+٤ if a٢n+١ in the series for f(x) is of order n'. From equation (١٢) it can be seen immediately that this inequality supplies a necessary and sufficient condition that dz/dt, the rate of denudation, Mr. E. H. CULLING al cos e ٢١ ١ X e,K(٢n+l)٢٢t/ ٤١٢ for all x, ٠ < x < ١, between the absolute value of the first term of series (١٦) and that of the sum of the remaining terms is true for all t > to, where to depends mainly upon the behavior of the exponential factors but also upon the nature of the coefficients a٢n+ and therefore upon the initial profile. Derivatives of series (١٦) of all orders with respect to x and t exist for all x, ٠ K x < ١, and all to, ٠ < to < T. A relation of similar form to expression (١٩) holds for each, though the value of two required will be greater the higher the order of the derivative. The first term of the series does not change sign in ٠ < x < l; and if the coefficient a١ is positive, as will clearly be the case in all applications to denudation, the restriction to the absolute value can be relaxed on the left-hand side of expression (١٩). Then the value of the gradient f'(x) will eventually become negative throughout the region ٠ < x < ١, as will the curvature"(x). The third and fourth derivatives will eventually become positive throughout the region. As K is a positive constant, both sides of equation (١٢) will eventually become negative for all x, ٠ < x < ١, so that for the rate of denudation dz/dt to decrease with time we require d٢z/dt٢ > ٠, which in turn implies will decrease in absolute value throughout the region ٠ x < ١, occupied by the slope profile from watershed to base level, and also from expressions (١٨) and (١٩) that this condition will eventually be fulfilled in all cases whatever the initial profile. This is a far less restrictive condition than the former involving the disappearance of all terms in the cosine series other than the fundamental. It will be fulfilled at or before the time when the relation ٧rx al' cos eK"'t/٤١٢ * , (٢n+I)ir x > n+ COs٢١nl١ X eK(٢n+)٢wt/٤ (٢١) _ cos (٢nI+)x is satisfied for all x, ٠ < x < ١. The least favorable case will be that for which at any point within the interval (٠, ١) the series on the right-hand side is made up entirely of negative terms, thus rendering the two sums equivalent and making it necessary to wait until expression (٢١) is satisfied under the most stringent conditions. But this, in virtue of relation (١٨), is still some time before the right-hand side can be taken as of negligible value, as required by the former condition. To interpret the condition in a manner more suited to practical use, recall that, for a slope profile with stable base level, denudation proceeds according to equation (١٢) and is directed toward a curve, regular and convex to the sky, roughly of cosine form (٠, r/٢). As soon as the regularization of the profile has reached the stage where the curvature f"(x) is a regular negative strictly monotonic function, increasing from a minimum at the watershed to a maximum at the point of local base level, and with a graph concave to the x-axis (roughly of cosine form [Tr, ٣٧r/٢]), the derivative f"(x) will necessarily be an increasing function in the interval (٠, ١), roughly of sine form (٠, ٧r/٢), and five(x) is necessarily positive. Thus for practical purposes a slope profile with a convextothesky curve and a negative curvature increasing steadily from watershed to base level will be subject to a decreasing rate of denudation throughout its length apart from the point at base level and will continue so provided there is no interference with the system from outside. The convexity condition needs to be introduced somewhere, as it is essential for a complete statement but is redundant physically.

If the second derivative is negative and a strictly monotonic increasing function but is convex to the x-axis, the third derivative is a decreasing function and the fourth negative. However, this also implies that f(x) is negative and increasing in the interval (٠, ١) from watershed to base level. The practical importance of the less restrictive condition is soon apparent in the field. Whereas cosine curves are few and far between in the soil covered landscape, convex profiles with a steadily increasing curvature toward base level abound. Where the initial profile is not symmetrically disposed about the watershed or where the local base levels on either side of an interfluve, though stable, are at different elevations, then, in the usual terminology, the divide migrates. In this case the cosine series is best replaced by a sine series, and the interval needs to be taken from one base level at x = ٠ to the other at x = ١. If the elevations are, respectively, zl and z٢, the elevation of the interfluve for t > ٠ is x z= Zl+ (z٢ Z) ٢ Zl cos nTr zi . nrx +Z sin nl n ١ ٢ nirx X eKn٢٢ t٢ Sin n=l (٢٢) X eKn٢t/٢ f (x') sin dZx' Jo (Carslaw and Jaeger, ١٩٥٩, p. ١٠٠, ٣.٤ [١]). The first two terms represent the steady state solution, that which will ultimately prevail. The next term comprises a sine series equal and opposite to the steady state solution with an exponential factor that rapidly weakens its value with time, so permitting the steady state terms to attain their full effect. The final term represents the waning effect of the initial profile f(x'). Similar relations hold with respect to the sine series as those established for the cosine series; eventually the fundamental term becomes dominant, the other terms vanishing, so that, as far as the initial profile is concerned, the divide will by that time (if not before) have migrated to a point midway between the two bounding base levels. Meanwhile the effect of the differing elevations on either side of the interfluve will be making itself felt, tending to deflect the divide toward the base level with the greater elevation. From the nature of the sine curve, deflection will become increasingly difficult, greater and greater differences in the base levels being required to produce additional increments of deflection. While the divide is migrating, the possibility arises of points of the slope profile in advance of the divide suffering an increase in the absolute value of the curvature, so becoming subject to an increased rate of denudation. Migration may be so slight, however, that the overall decrease in amplitude with time will offset any tendency for the curvature to increase in absolute value. Migration due to differing base levels, if stable, probably falls within this category, unless the difference is excessive. More rapid migration is likely to arise from the form of the initial profile; but as it will be concentrated in youth or early maturity at the latest, before the trend to decreased rates of denudation has been completely established, its significance will be minimal. Nevertheless the migration of divides is a mild form of rejuvenation, and the neighborhood of the divide may provide an exception to the general rule of decreasing rates of denudation in soil covered landscapes subject to stable base levels. If the local base level is not stable but either rises or migrates laterally away from the foot of the slope or although stable is separated from the slope by a landform of lower gradient, one section at least of the lower part of the profile from watershed to base level presents a concavetothesky curve, or if irregular, at least one reentrant angle. Taking the initial position of base level as datum, the elevation f(x) of the profile from watershed to base level is everywhere positive or zero and the slope gradient f'(x) everywhere negative or zero. Any minor section of positive gradient will be rapidly eliminated; any major section demands a redefinition of the extent of the slope profile. But the curvature f"(x) of the profile will include at least one positive section, one negative, and at least one point of zero value, provided we agree to regard sharp breaks of slope as limiting cases and where necessary the watershed and base-level points as points of negative and positive curvature, respectively. As K in equation (١٢) is a positive constant, the direction in the vertical plane taken by a point on the landform surface is of the same sign as the curvature at that point, so that sections of positive curvature are subject to deposition. The analysis already given for the case of denudation only can be extended to cover deposition, changing to a sine series or to an interval (٠, ٧r) if necessary. A relation similar to (١٩) will eventually be satisfied in almost all cases, but the time interval required depends to a far greater degree upon the form of the initial profile. Denudation alone is concerned with the reduction of a given landmass; but, if deposition enters the picture, the space available and upon which derived material or a portion is to be deposited must also be considered. Deposition implies a closed or partly closed system, that is, the lower boundary control is at least partly reflexive. Where the landmass occupies a small portion of the available area, the elimination of the initial surface may require a lengthy period in comparison with the complete cycle. At the limit the controlling local base level is removed to infinity, and the time required is coextensive with the period for the completion of the cycle to perfection which is infinitely long. Physically, if the local base level is separated from a landform by a surface of low gradient and of sufficient extent to render it in effect infinitely remote, the position and behavior of the base level will have no influence upon the reduction of the landmass and the deposition of the derived material which will be spread ever more widely and thinly. Not only is the direction taken by a point on the landform surface of the same sign as the curvature, the rate of movement is proportional to the magnitude of the curvature. During the reduction of a landscape by soil creep the tendency is always to decrease the absolute value of the profile curvature at points at which it takes maximal or minimal values. Once the influence of the initial profile has been eliminated and the slope profile is a simple regular sigmoid curve, or, more precisely, once the curvature f"(x) is a regular strictly monotonic increasing function, ranging from negative to positive values from watershed to base level, then the absolute value of the curvature will decrease for all points of the profile with the exception of the neighborhood of the point of inflection which may migrate owing to the influence of spatial restrictions placed upon deposition. The former and more general statement almost ranks as the equivalent within the realm of geomorphology of Whittaker's charming postulates of impotence, though it lacks the grand cosmical manner. It may be reworded as the impossibility of finding a soil covered slope with a greater maximum absolute value to the profile curvature than its predecessor, unless there has been outside interference. From this statement it is possible to deduce the dependence of the rate of denudation upon the profile curvature and of material transport on the slope gradient, and all else follows. It is not possible, however, to deduce a statistical law from a set of deterministic equations so that the random displacements of soil particles are not a necessary consequence and can be supplanted by any other hypothesis leading to a diffusivity type of equation. Like the postulates of impotence the statement concerning the surface curvature is not a logically necessary result but is ultimately grounded in our experience of the real world. It is not only conceivable that a landscape could develop according to some other principle; it is a matter of observation that slopes without a soil cover actually does so. Of all natural phenomena within the realm of the earth sciences one of the most remarkable is the flowing regularity of soil coveredlandscapes. A commonplace observation, nevertheless it suggests a summary of the whole of the Davisian scheme of humid erosion in one sentence: the soil covered landscape is always tending to straighten out its curvature. The remainder of the Davisian scheme can be regarded as interference with this general tendency. It is to this aspect that we finally turn. Accidental disturbances to the normal course of erosion on soil covered slopes include environmental change of parent rock, climate, or vegetation; variation in runoff and percolation rates; and interference both natural and artificial with the soil, vegetation, or drainage. These disturbances operate through the medium of the surface cover to limit or terminate the activity of soil creep as a denudational agent. More important from our point of view are the systematic influences associated with the progress of the erosion cycle. Broadly they arise from the abstraction of too much or the supply of too little material to the system. Depletion in the supply of material to the soil cover is liable to occur for the very good reason that there is none to supply. At a watershed all flow is away from the summit. The soil thickness is diminished, and despite an accelerated rate of weathering it may fall below the value required for unimpaired soil creep. The intensity of the effect depends upon the curvature of the surface and therefore increases up to maturity when watersheds become the points of maximal curvature. Thereafter the rate of denudation declines so that, apart from the question of divide migration, a summit point that has retained a soil cover up to the stage of maturity will continue to do so throughout the remainder of the cycle. As the summit point suffers most, if that point is adequately covered, then so will be the rest of the slope. Conversely, it is at the onset of maturity and at summit points that a soil covered slope is most prone to attenuation and its deleterious effect upon soil creep. Excessive abstraction of material from the soil cover follows dissection of the landscape and the lowering of local base levels. Drainage systems impose a network of time-variable boundary conditions to which the landforms attempt to conform. Lack of success is heralded by the impairment of soil creep worsening to breakdown and loss of continuity in the lowest sections of the surface cover. The immediate effect of a falling base level or of one that laterally undercuts is to steepen the adjacent slope so producing a local increase in the rate of change of gradient. The normal activity of soil creep will tend to eliminate this local sector of increased curvature and in effect dissipates the irregularity throughout the profile. A persistent fall of base level will continually present a freshly steepened section at the base of the slope superimposing on the normal degradational pattern of decreasing gradient and increasing curvature away from base level and both decreasing at all points with time, a rejuvenation pattern in which both gradient and curvature decrease away from maxima at base level and both tend to increase at all points with time. The elevation during rejuvenation of the soil covered interfluve in the region ٠ < x < l subject to variable base levels at x = ٠ and x = l is given by ٢ ٢ . mrx z= eKnt/٢٢ sin n= ١ [f . nx' (٢٣) X [f(x')sinm ddx' + nrK o Kn٢r٢/١٢Kn t١(X) (I)"P٢(X) } dX] ٢٥٢ (Carslaw and Jaeger, ١٩٥٩, p. ١٠٤ ٣.٥ [٢]), where ٤l(t) and q٢(t) are the elevations at x = ٠ and x = ١, respectively, and f(x') is the initial profile. Taking the simplest case, that in which both local base levels fall at a constant rate kt from the same initial elevation, taken as zero, we have k (x٢ xl) ٤k١٢ z = kt+ + ٢K Kr٣ I . (٢n+I)rx (٢٤) x Z(٢n+I) sn X eK(٢n+I)rst/l٢ plus a term with respect to the initial profile. Ultimately the profile is given by the first two terms whatever the initial profile, provided both local base levels maintain identical elevations. The curvature f"(x) of the interfluvial profile will then be a constant; and, as is to be expected, the form of the profile is timeinvariant. Thus with a constant rate of fall of base level the two patterns cancel out; an increasing rate will impose the rejuvenation pattern; a decreasing rate will fail to obliterate the normal degradational pattern. Expression (٢٣) covers all possible cases of rejuvenation by vertical fall of local base level. If necessary, ٤(t) can be represented by a Fourier series, but the solution is likely to be cumbersome. Unequal initial elevations of the two base levels will lead to migration of the divide from the midpoint in youth; unequal rates of fall lead to continued migration in post maturity. When the rate of rejuvenation slackens, the normal degradational pattern is gradually reimposed; the point of maximal absolute curvature migrates away from the point at base level, preceded by a region of rising absolute values and followed by a wave of declining values, until it finally reaches the watershed. The normal course of denudation on soil covered slopes proceeds from diversity to uniformity, from the irregular to the regular, while the soil cover proceeds from order to disorder, from the less probable to the more probable state, and may loosely be described as the gradual increase in entropy within the system of the reducible mass of the landscape. Rejuvenation imposes local increases in the concentration gradient; progress to uniformity is temporarily reversed; order is increased; the system changes into a less probable state. Broadly the potential energy of the system is raised by the increase in relief. Surface elevation plays a similar role to that of temperature (Culling, ١٩٦٠), and the thermodynamic analogies proposed by Leopold and Langbein (١٩٦٢) are valid in principle, at least so far as soil covered landscapes are concerned. NONSOILCOVERED SLOPES On a steepening slope, long before the critical changeover from control of denudation by the rate of material transport to control by the rate of weathering, the soil structure breaks down, with loss of cohesion and retentivity. Vegetation fails, surface flow is permitted, and protection against raindrop impact is lost. Accelerated forms of erosion appear in isolated patches but have no great influence upon the course of denudation until they have spread to coalesce and cover the greater part of the surface. But the accelerated rate of transport need not necessarily be sufficient to outstrip the supply of material and so bring about a change in the whole denudational regime. The changeover is a complex process, depending upon the interrelationship of various factors and is not a critical point but a broad transition zone between the two modes of denudation. The agents of weathering attack the rock surface normally (Scheidegger, ١٩٦٠b, p. ٢٠٣), and in homogenous material provide equal amounts of weathered material during equal periods for equal areas, provided that the weathered material is removed equally or as soon as it is produced and that the surface is a plane. Otherwise accumulating material will slow the rate of weathering, and unequal accumulation will result in unequal rates of weathering. The unfettered action of weathering places lithological qualities at a premium, the surface following every stratum and reflecting every variation in resistance. The structural control of the landscape often verges on the fantastic. Systematic treatment is out of the question for a large proportion of such terrains. It is only where lithological uniformity imparts some degree of regularity that slopes subject to control by weathering become amenable to mathematical treatment. The model proposed by Scheidegger (١٩٦١a) covers a wide range of slopes for which the rate of weathering is the controlling factor. Weathering acting normal to the surface has an effect upon the slope elevation varying with the secant of the slope angle. Thus = /V[I+ (y/x)٢], where ٤ is a factor qualifying the weathering action and representing the protection afforded by the layer of weathered material in transit. Clearly this factor, like the one qualifying the diffusion coefficient in equation (١٤), ranges from zero to unity. The precise behavior will be difficult to determine. Scheidegger proposes a variation with the surface gradient, giving = [I + (Oy/Ox)٢]. ct ٣x For the application of this equation the reader is referred to a series of papers by Scheidegger outlining the model (١٩٦١a) and then applying it to undercut slopes (١٩٦٠b). Various qualitative theories are discussed and compared with the predictions of the model (١٩٦١b). Subsequently the model has been generalized to include the ideas of Bakker and Le Heux (١٩٦٢a) and has also been applied to marine terraces (١٩٦٢b).

REFERENCES CITED

CARSLAW, H. S., ١٩٣٠, Introduction to the theory of Fourier series and integrals: London, Macmillan Co. and JAEGER, J. C., ١٩٥٩, Conduction of heat in solids, ٢d ed.: London, Oxford University Press. CRAMER, H., ١٩٦٢, Random variables and probability distributions, ٢d ed.: London, Cambridge University Press. CRANK, J., ١٩٥٦, Mathematics of diffusion: London, Oxford University Press. CULLING, W. E. H., ١٩٦٠, Analytical theory of erosion: Jour. Geology, v. ٦٨, p. ٣٣٦٣٤٤. ١٩٦٣, Soil creep and the development of hillside slopes: ibid., v. ٧١, p. ١٢٧١٦١. EINSTEIN, A. ١٩٢٦, Investigations on the theory of the Brownian movement, trans. A. D. COWPER, Dover ed. (١٩٥٦): New York, Dover Publications. FURTH,R ., ١٩٢٦, Notes to EINSTEIN, A., Investigations on the theory of the Brownian movement: New York, Dover Publications. JOST, W., ١٩٥٢, Diffusion in solids, liquids and gases: New York, Academic Press. LEOPOLD, L. B., and LANGBEIN, W. B., ١٩٦٢, The concept of entropy in landscape evolution: U.S. Geol. Survey Prof. Paper ٥٠٠A. MELTON, M. A., ١٩٥٨, Geometric properties of mature drainage systems and their representation in an E٤ phase space: Jour. Geology, v. ٦٦, p. ٣٥٥٤. MISES, R. VON, ١٩٥٧, Probability, statistics and truth, ٢d rev. English ed.: London, George Allen & Unwin. POPPER, K. R., ١٩٥٩, The logic of scientific discovery: London, Hutchinson. RUSSELL, E. J., ١٩٥٠, Soil conditions and plant growth, ٨th ed.: London, Longmans Green & Co. SCHEIDEGGER, A. E., ١٩٦٠a, Physics of flow through porous media, ٢d ed.: Toronto, University of Toronto Press. ١٩٦٠b, Analytical theory of slope development by undercutting: Alberta Soc. Petroleum Geologists Jour., v. ٨, p. ٢٠٢٢٠٦. ١٩٦١a, Mathematical models of slope development: Geol. Soc. America Bull., v. ٧٢, p. ٣٧٥٠. ١٩٦١b, Evaluation of slope development theories: Alberta Soc. Petroleum Geologists Jour., v. ٩, p. ١٥١٩. ١٩٦١c, On the statistical properties of some transport equations: Can. Jour. Physics, v. ٣٩, p. ١٥٧٣١٥٨٢. ١٩٦١d, General theory of dispersion in porous media: Jour. Geophys. Research, v. ٦٦, p. ٣٢٧٣ ٣٢٧٨. ١٩٦٢a, Some modifications of the slope development problem: Royal Astron. Soc. Geophys. Jour., v. ٧, p. ٤٠٥٣. ١٩٦٢b, Marine terraces: Geofisica Pura e Applicata [Milan], v. ٥٢, p. ٦٩٨٢. ٢٥٤