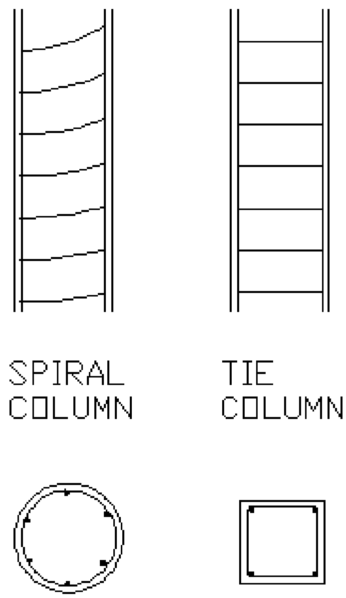
Design of reinforced concrete columns

Type of columns



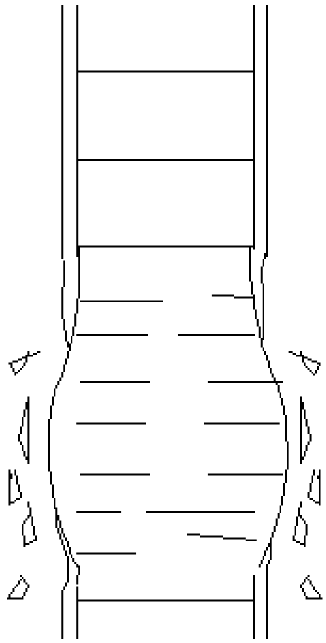
Failure of reinforced concrete columns

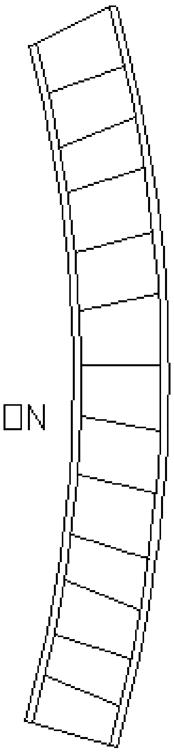
Short column

Column fails in concrete crushed and bursting. Outward pressure break horizontal ties and bend vertical reinforcements

Long column

Column fails in lateral bucklin





See test picture from web-site below

See picture from web-site below

part-3.html

struct-walls.htm

Short column or Long column?

ACI definition

For frame braced against side sway:	For Frame not braced against side sway:
Long column if $kl_u/r > 34-12(M_1/M_2)$ or 40	Long column if $kl_u/r > 22$

Where k is slenderness factor, l_u is unsupported length, and r is radius of gyration. M1 and M2 are the smaller and larger end moments. The value, (M_1/M_2) is positive if the member is bent in single curve, negative if the member is bent in double curve.

Determine the slenderness factor, k

The slender factor, k should be determined graphically from the Jackson and Moreland Alignment charts.

(Charts will be added later)

where ψ E($\sum$ $={}_c I_c/l_c$) of column /$\sum$ E$({}_b I_b/l_b)$ of beam, is the ratio of effective length factors.

E_c and E_c are younger modulus of column and beams.

l_c and l_c are unbraced length of column and beams.

The cracked moment of inertia, I_c is taken as 0.7 times gross moment of column and I_b is taken as 0.35 times gross moment of inertia of beam.

Alternatively, k can be calculated as follows:

1. For braced frame with no sway,

k can be taken as the smaller value of the two equations below.

$k = 0.7 + 0.05\ (\psi_A+\psi_B) \leq ,1$

$k = 0.8 + 0.05\ (\psi_{min}) \leq 1$

ψ_A and ψ_B are the ψ at both ends, ψ_{min} is the smaller of the two ψ values.

2. For unbraced frame with restrains at both ends,

For $\psi_m < 2$

$k = [(20- \psi_m)/20] \sqrt{(1+\psi_m)}$

For $\psi_m \geq 2$

$k = 0.9 \sqrt{(1+\psi_{min})}$

ψ_m is the average of the two ψ values.

2. For unbraced frame with restrain at one end, hinge at the other.

$k = 2.0 + 0.3 \psi$

ψ is the effective length factor at the restrained end.

Example:

Beam information:

Beam size: b = 18 in, h = 24 in

Beam unsupported length: l_b = 30 ft

Concrete strength: 4000 psi

Young's modulus, E_b = 57 0004√ 5063 = ksi

Moment of inertia of beam: $I_b = 0.35bh^3/12 = 7258$ in^4.

Column information:

Square Column: D = 18 in, unsupported length l_c =10 ft

Concrete strength: 5000 psi

Young's modulus: E_c = 57 0005√ 0304 = ksi

moment of inertia of column: $I_c = 0.7D^4/12 = 6124\ in^4$.

Column top condition:

There are beams at both sides of column at top of column, no column stop above the beams

The effective length factor: ψ_A E(=$_c I_c/l_c$) /[2 E($_b I_b/l_b$)] = 1.4

Column bottom condition:

There are beams at both sides of column at bottom of column and a column at bottom level

The effective length factor: ψ_A E(2[=$_c I_c/l_c$)] / [2 E($_b I_b/l_b$)] = 2.8

From chart:

If the column is braced: k ≈ 0.84

If the column is unbraced: k ≈ 1.61

From equation

If the column is braced:

$k = 0.7 + 0.05\ (\psi_A+\psi_B) =$ 19.0

$k = 0.8 + 0.05\ (\psi_{min}) =$ 29.0

If the column is unbraced: $\psi_m = (\psi_A+\psi_B)/2 = 2.12$

$k = 0.9\ \sqrt{(1+\psi_{min})} = 1.6$

Design of reinforced concrete columns

Short column	Long non-sway column & Long sway columns

1. Column shall be designed to resist factored axial compressive load and factored moments.

2. Column strength shall be determined based on strain compatibility analysis.

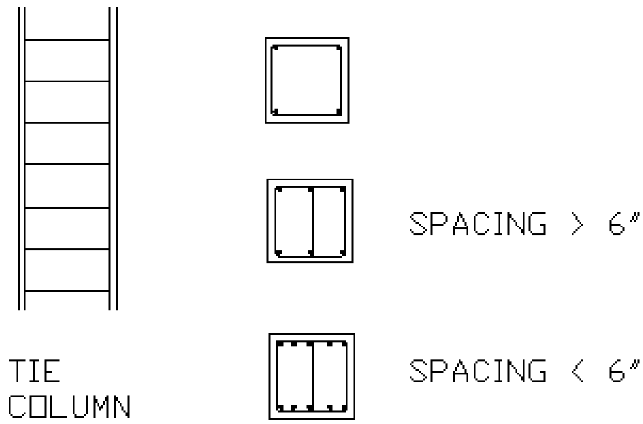
1. Column shall be designed to resist factored axial compressive load. Factored moment shall be magnified with magnification factors.

2. Column strength shall be determined based on strain compatibility analysis.

Column ties and spiral

ACI code requirements for column ties

1. No. 3 ties for longitudinal reinforcement no. 10 bars or less, no. 4 ties for no. 11 bars or larger and bundled bars.
2. Tie spacing shall not exceed 16 diameter of longitudinal bars, 48 diameters of tie bars, nor the least dimension of column.
3. Every corner bar and alternate bars shall have lateral tie provide the angle shall not exceed 135 degree.
4. No longitudinal bar shall be spacing more than 6 inches without a lateral tie.



ACI code requirements for spiral

1. Sprial shall be evenly space continuous bar or wire, no. 3 or larger.
2. Sprial spacing shall not exceeds 3 in, nor be less than 1 in.
3. Anchorage of spiral shall be provided by 1-1/2 extra turn.

Design of short columns

Design of long non-sway columns

Design of long column with sway

Design of short concrete columns

Strength of column subjected to axial load only

Ideally, if a column is subjected the pure axial load, concrete and reinforcing steel will have the same amount of shortening. Concrete reaches its maximum strength at $0.85f_c'$ first. Then, concrete continues to yield until steel reaches its yield strength, f_y, when the column fails. The strength contributed by concrete is $0.85f'_c(A_g-A_{st})$, where f_c' is compressive strength of concreter, A_g is gross area of column, A_{st} is areas of reinforcing steel. The strength provided by reinforcing steel is $A_{st}f_y$. Therefore, the nominal strength of a reinforced concrete column, is

$$P_n = 0.85f'_c(A_g-A_{st})+A_{st}f_y \quad [1]$$

For design purpose, ACI specify column strength as follows

For a spiral column, the design strength is

$$\phi P_n = 0.85\phi[0.85f'_c(A_g-A_{st})+A_{st}f_y] \quad [2]$$

For a regular tie column, the design strength is

$$\phi P_n = 0.80\phi[0.85f'_c(A_g-A_{st})+A_{st}f_y] \quad [3]$$

where ϕ is strength reduction factor. For spiral column ϕ 57.0 = (ACI 318-99), ϕ 7.0 = (ACI 318-02, 05). For spiral column ϕ 7.0 = (ACI 318-99), ϕ 56.0 = (ACI 318-02, 05)

The factors 0.85ϕ and 0.8ϕ are considering the effect of confinement of column ties and strength reduction due to failure mode. Nevertheless, column loads are never purely axial. There is always bending along with axial load.

Strength of column subjected to axial load and bending

Consider a column subjected to axial load, P and bending moment, M. Axial load, P produces an uniform stress distribution across the section while bending moment produces tensile stress on one side and compressive stress on the other.

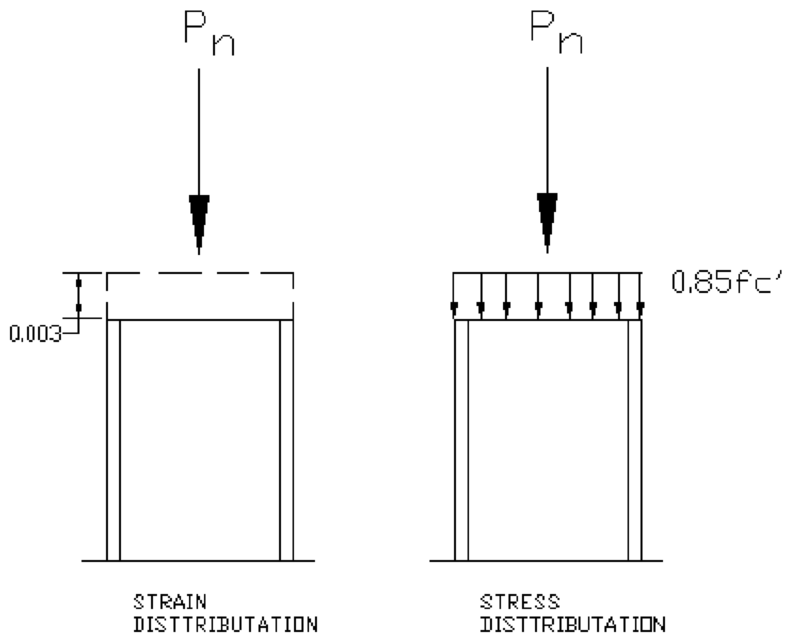
Strain and stress distributions of short concrete column at failure and interactive diagram

Assumption:

1. A plan section remains a plan at failure. Strain distributes linearly across section
2. Concrete fails at a strain of 0.003.
3. Reinforcing steel fails at a strain of 0.005.

Axial load only (moment = 0) **Moment onl**

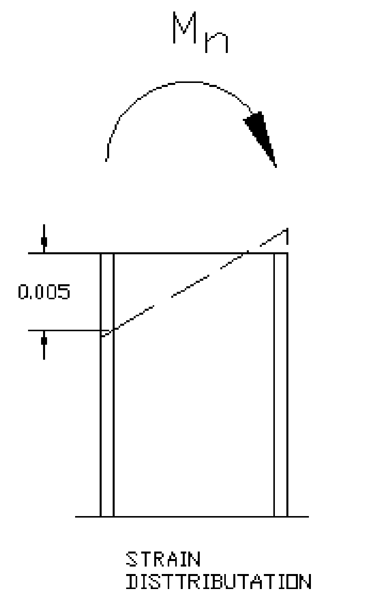
Failure occurs when concrete strain reaches 0.003



Large axial load with small moment

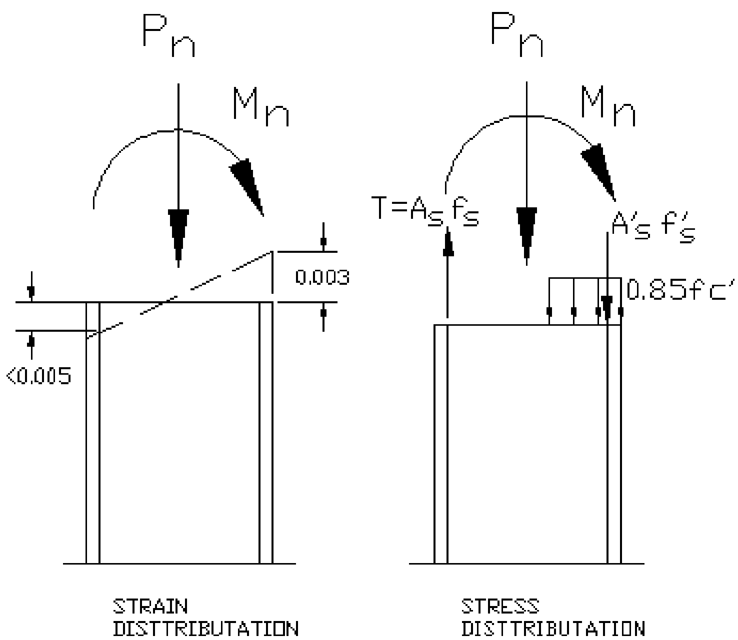
Failure occurs when concrete strain reaches 0.003

Failure occurs when ste



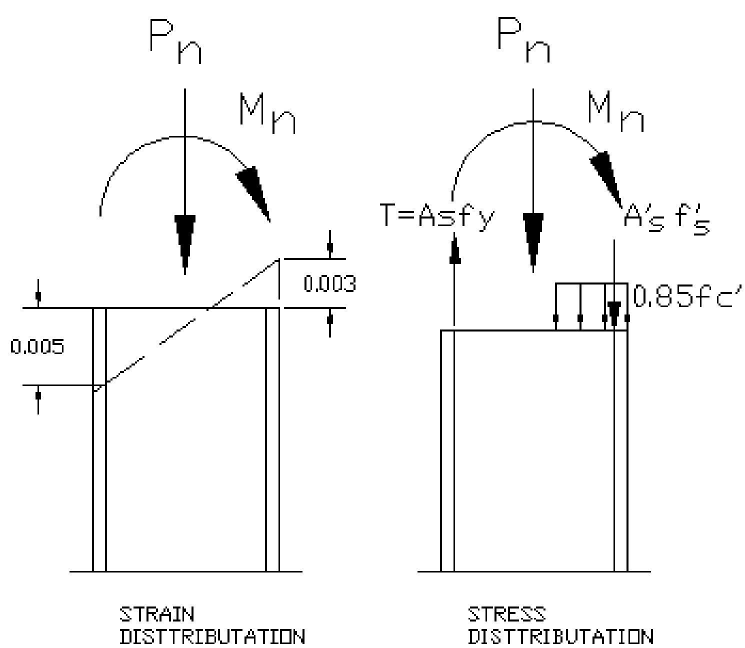
Small axial loa

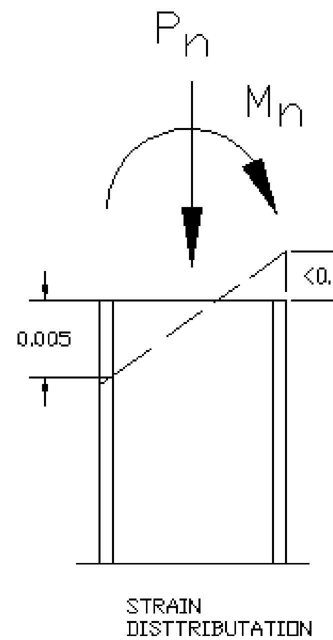
Failure occurs when



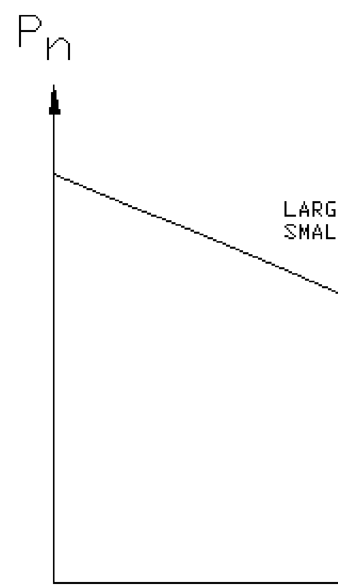
Balanced condition

Failure occurs when concrete strain reaches 0.003 and steel strain reaches 0.005 at the same time.



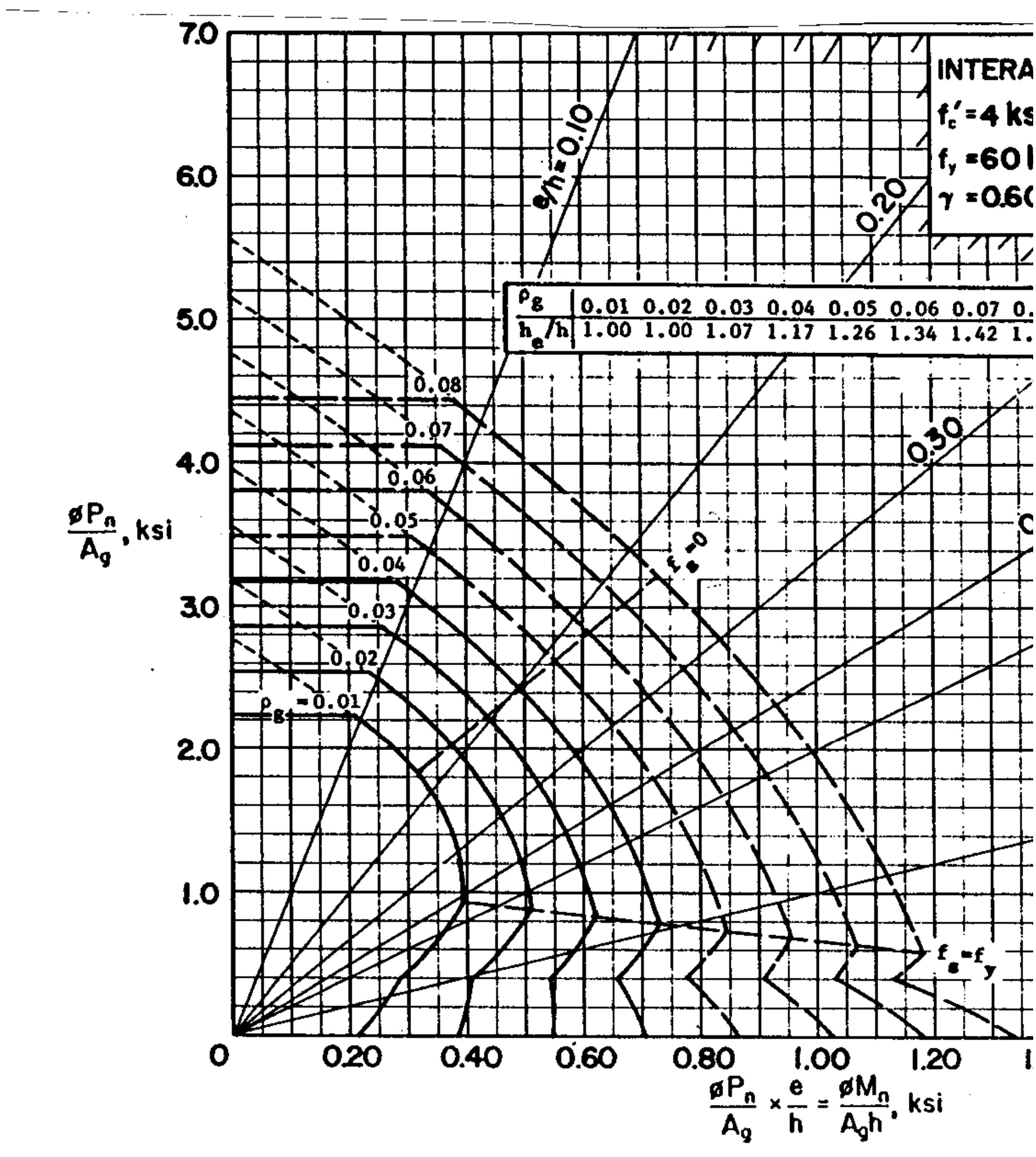


Interaction dia



Design aid:

The interaction diagrams of concrete column with strength reduction factor is available on ACI design handbook. The vertical axis is $\phi P_n / A_g$ and the horizontal axis is $\phi M_n / A_g h$, where h is the dimension of column in the direction of moment. The chart is arranged based on the ratio, γ which is the ratio of the distance between center of longitudinal reinforcements to h.



Column strength interaction diagram for rectangular column with γ =0.6 (Coursey of American Concrete Institute)

ACI design handbook can be purchase from ACI book store. The title is "SP-17: Design Handbook: Beams, One-Way Slabs, Brackets, Footings, Pile Caps, Columns, Two-Way Slabs, and Seismic Design in accordance with the Strength Design Method of 318-95"

Design of short concrete column

Design requirements:

1. Design strength: $\phi P_n \geq P_u$ and $\phi M_n \geq M_u$
2. Minimum eccentricity, $e = M_u/P_u \geq .1.0$

Design procedure:

1. Calculate factored axial load, P_u and factored moment, M_u.
2. Select a trial column column with b and column depth, h in the direction of moment.
3. Calculate gross area, A_g and ratio, γ = distance between rebar/h.
4. Calculate ratio, P_u/Ag and $M_u/A_g h$.
5. Select reinforcement ratio,,$\rho$from PCA design chart based on concrete strength, f_c', steel yield strength, f_y, and the ratio, γ.
6. Calculate area of column reinforcement, A_s, and select rebar number and size.
7. Design column ties.

Design example:

Example: A 12"x12" interior reinforced concreter short column is supporting a factored axial load of 350 kips and a factored moment of 35 kip-ft.

Desogn data:

Factored axial: P_u = 350 kips

Factored moment: M_u = 35 ft-kips

Compressive strength of concrete: f_c'= 4000 psi

Yield strength of steel: f_y = 60 ksi

Requirement: design column reinforcements.

Column size: b = 12 in, h = 12 in

Gross area, Ag = 144 in^2.

Concrete cover: d_c = 1.5 in

Assume #4 ties, d_t = 0.5 in and #6 bars, d_s = 0.75 in

Calculate γ ($=h - 2\, d_c - 2\, d_t - d_s)/h = 0.6$

Calculate,

$P_u/A_g = 300/144 = 2.43$ ksi,

$M_u/A_gh = 35/[(144)(12)] = 0.243$

From ACI design handbook, reinforcement ratio, ρ 810.0 =

Area of reinforcement, $A_s = (0.0018)(144) = 2.6$ in^2.

Use 6#6, area of reinforcement, $A_s = (6)(0.44) = 2.64$ in^2.

Check Bar spacing, s = (h - 2 dc - 2 dt - ds)/2 = 3.625 in (O.K.)

Calculate minimum spacing of column ties:

48 times of tie bar diameter = (48)(0.5) = 24 in

16 times of longitudinal bar diameter = (16)(0.75) = 12 in

Minimum diameter of column = 12 in

Use #4 ties at 12 inches on center.

Design of long column in non-sway frame (ACI 318-02,05)

Moment magnification for columns in braced frames (non-sway)

For a slender column in a braced frame that is subjected to axial compression and moments. If M_1 is the smaller and M_2 is the larger moment, the moment need to be design for magnified moment if the ratio

$kl_u/r > M(21 - 43_1M/_2)$

where k is slenderness ratio, l_u is unsupported length, r is radius of gyration. k shall not be taken as 1 unless analysis shows that a lower value is justified.

$M_1M/_2$ is positive if the column is bent in single curve and $M_1M/_2$ is not to be taken less than -0.5 ($kl_u/r = 40$).

The design moment shall be amplified as

$M_c = \delta_{ns}M_2$

where

$\delta_{ns} = C_m/(1-P_u/0.75P_c) \geq 1$ is moment magnification factor for non-sway frame,

$C_m = 0.6+0.4(M_1/M_2) \geq 4.0$

P_u is factored column load, and

$P_c = \pi^2(/IEkl_u)^2$ is Euler's critical buckling axial load,

EI shall be taken as

$EI = (0.2E_cI_g+E_s/I_{se})/(1+\beta_d)$ or $EI = 0.4E_cI_g/(1+\beta_d)$

Where is the ratio of maximum factored axial dead load to total factored load

Example:

A 12"x12" interior reinforced concrete column is supporting a factored axail dead load of 200 kips and a factored axial live load of 150 kip. Factored column end moments are -35 kip-ft and 45 kip-ft. The column is a long column and has no sway.

Design data:

Total Factored axial: P_u = 350 kips

Factored moment: M_1 = -35 kips, M_2 = 45 kips (bent in single curve)

Compressive strength of concrete: 4000 ksi

Yield strength of steel: 60 ksi

Unsupported length of column: 10 ft

Requirement: Determine the magnified design moment

Column size: b = 12 in, h = 12 in

Gross area: Ag = 144 in^2.

Concrete cover: 1.5 in

Assume #4 ties and #8 bars, d_t = 0.5 in, d_s = 1 in

Gross moment of inertia: I_g = $(12 in)^4/12$ = 1728 in^4.

Radius of gyration, r = √(1728/144) = 3.5 in or r = 0.3(12 in) = 3.6 in

Assume slenderness factor, k = 1 without detail analysis

Slenderness factor, klu/r = (1)(120 in)/3.6 in = 35 > 34-12(35/45) = 25, long column

Young's modulus of concrete, E_c = 570004√ 5063 = ksi

Elastic modulus of steel: E_s = 29000 ksi

Assume 1% area of reinforcement: As = (0.01)(144 in^2) =1.44 in^2.

Assume half of the reinforcement at each side of column, distance between rebas = 12-1.5*2-0.5*2-1 = 7 in

Moment of inertia of steel reinforcement: I_{se} = (0.72 in^2)(7/2)^2*2 = 17.6 in^4.

The ratio of factored load, β_d = 200/350 = 0.57

The flexural stiffness, EI = 0.4$E_c I_g$/(1+β_d)= 0.4(3605)(144)/(1+0.57) = 1.58x10^6 kip-in^2.

or EI = (0.2$E_c I_g$+E_s/I_{se})/(1+β_d) 11.1 =)75.0+1(/])6.71()00092(+)441()5063(2.0[=x10^6 kip-in^2.

The critical load, P_c = π^2(/IEkl_u)^2 = π^2(1.58x10^6)/(35)^2 = 1087 kip

Factor Cm = 0.6+0.4(35/45) = 0.911

Moment magnification factor, δ_{ns} = C_m/(1-P_u/0.75P_c) = (0.911)/[1-350/1087] = 1.6

The magnified design moment, M_c = 1.6 (45) = 71.8 ft-kip

Design of long column in sway frame

Moment magnification for columns in unbraced frames (sway)

Determine if the frame is a sway frame

1. The frame can be assumed as non-sway if the end moment from second-order analysis not exceeds 5% of the first-order end moment.

2. The frame can be assumed as non-sway if stability index,

$Q = \sum P_u \Delta_o / V_u l_c \geq 50.0$

where $\sum P_u$ and V_u are the total vertical load and the story shear, Δ_o is the first-order relative deflection between the top and bottom of that story due to V_u, and l_c is the length of the column.

3. The moment need not to be design for magnified moment if the ratio

$kl_u/r \leq 22$

where k is slenderness ratio, l_u is unsupported length, r is radius of gyration.

Calculating magnified moments

If M_1 is the smaller and M_2 is the larger moment, the design moment shall be amplified as

$M_1 = M_{1ns}+\delta_s M_{1s}$

$M_2 = M_{2ns}+\delta_s M_{2s}$

where M_{1ns} and M_{2ns} are moments from loadings that are not contribute to sway (i.e. gravity load), and M_{2s} and M_{2s} are moments from loading that contribute to sway (i.e. wind and seismic)

1. The magnified sway moment $\delta_s M_s$ can be determined by a second-order analysis based on the member stiffness reduced for crack section.

2. The magnified sway moment can be calculated as

$\delta_s M_s = M_s/(1-Q) \geq M_s$ when $1 \leq \delta_s \leq 5.1$

3. The magnified sway moment can be calculated as

$\delta_s M_s = M_s/[1-\sum P_u/(0.75\sum P_c)] \geq M_s$

where $\sum P_u$ is the summation for all the factored vertical loads in a story and $\sum P_c = \pi \sum^2 (/IEkl_u)^2$ is the summation of critical bucking loads for all the columns in a story from non-sway frame.

Limitations

1. The ratio of second-order lateral deflection to the first-order lateral deflection based on factored dead and live loads plus lateral load shall not exceed 2.5.

2. The value of Q shall not exceed 0.6.

3. The magnification factor δ_s shall not exceed 2.5.

Example:

A reinforced concrete moment frame has four 18"x18" reinforced concrete columns.

Design data:

Factored column axial loads:

Column 1 & 4 (Exterior columns)

Live load: $P_{L1} = 150$ kips

Dead load: $P_{D1} = 200$ kips

Lateral load: $P_{W1} = 40$ kips

Column 2 & 3 (interior columns)

Live load: $P_{L2} = 300$ kips

Dead load: $P_{D2} = 400$ kips

Lateral load: $P_{W2} = 20$ kips

Factored column moments:

Column 1 & 4 (Exterior columns)

Live load moment: $M_{L11} = -25$ ft-kips, $M_{L12} = 40$ ft-kips

Dead load moment: M_{D11} = -35 ft-kips, M_{D12} = 45 ft-kips

Lateral load moment: M_{W11} = -75 ft-kips, M_{W12} = 80 ft-kips

<u>Column 2 & 3 (interior columns)</u>

Live load moment: M_{L21} = -15 ft-kips, M_{L22} = 25 ft-kips

Dead load moment: M_{D21} = -30 ft-kips, M_{L22} = 40 ft-kips

Lateral load moment: M_{W21} = -65 ft-kips, M_{W22} = 75 ft-kips

Compressive strength of concrete: f_c' = 4000 psi

Yield strength of steel: f_y = 60 ksi

Unsupported length of column: l_u = 10 ft

<u>Requirement</u>: Calculate magnification design moments for column 2 and 3.

1. Calculate total factored loads

Column 2 & 3: $P_u = (P_{L2}+P_{D2}+P_{W2})$ = 720 kips

All column:

$\sum P_u = (P_{L1}+P_{D1}+P_{W1}+P_{L2}+P_{D2}+P_{W2})$ = 2220 kips

Column size: b = 18 in. h = 18 in.

Gross area of column: Ag = bh = 324 in^2.

Assume concrete cover: 1.5 in for interior column

Assume #4 ties, d_t = 0.5 in and #8 bars, d_s = 1 in, for vertical reinforcement

Calculate gross moment of inertia of column: $I_g = bh^3/12$ = 8748 in^4.

Radius of gyration, $r = \sqrt{I_g/A_g}$ = 5.2 in

Assume slenderness factor, k =2 for moment frame with side-sway,

the slenderness ratio, k l_u/r = 46. > 22 (long column)

Young's modulus of concrete, E_c = 57 ()0004√ 5063 = ksi

Elastic modulus of steel: E_s = 29000 ksi

Assume 1% of reinforcement area, A_s = 0.01 A_g = 3.24 in^2.

Assume that reinforcement are placed equally at each side of the column, the distance between vertical reinforcement is

18-2(1.5)-2(0.5)-1 = 12 in

the moment of inertia of reinforcement, I_{se} = As $(13/2)^2$ = 137 in^4.

Factor, $\beta_d = P_{D2}/(P_{D2}+P_{L2}+P_{W2})$ = 0.556

The stiffness factor, EI = $(0.2E_cI_g+E_s/I_{se})/(1+\beta_d)$ 16.6 =x10^6 kip-in^2.

or EI = $0.4E_cI_g/(1+\beta_d)$ 11.8 =x10^6 kip-in^2.

Calculate magnification factor for non-sway moment:

$P_c = \pi^2(/IEkl_u)^2$ = 1390 kips.

Assume C_m = 1 (Transverse load at top and bottom of column)

$\delta_{ns} = C_m/[1-P_u/(0.75P_c\ )]$ = 3.235

Calculate magnification factor fo sway moment:

Euler's critical buckling axial load of all columns, $\sum P_c$ = $\pi^2(/IEkl_u)^2$ x 4 = 5558 kips.

The moment magnification factor, $\delta_s = 1/[1-\sum P_u/(0.75\sum P_c\ )]$ = 2.14

Magnified design for column 2 and 3, $M_{u2} = \delta_{ns} (M_{L22}+M_{D22})+\delta_s (M_{W2}) = 370.7$ ft-kip