

Standing Waves

**“Stationary, non-uniform Flow with considering of curvature
Filament “**

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1. Introduction:

With regard to the shape of the water level in undulating hydraulic jump is very little literature. Eisner ["open channels" in Vienna Harms: Handbook of Experimental Physics, Volume 4: Hydro-and Aeromechanics, Part 4] gives the differential equation of the stationary, non-

uniform motion without streamline curvature at $i - \frac{dT_m}{dx} = \frac{\lambda}{2g} \cdot \left(\frac{Wm^2}{\frac{F}{U}} \right) + \frac{d}{dx} \left(v \cdot \frac{Wm^2}{2g} \right)$. Without

further explanation, he goes (with reference to Boussinesq) into an abstract form of the differential equation of the Steady-state, non-uniform motion with streamline curvature and passes to the solutions. From his statement does not indicate, however, as the coefficients of the differential equations are determined.

It is therefore to determine how these coefficients are related to the terms of the flow.

Furthermore, the solution of the differential equation to a specific test is applicable.

2. Forming the main formula (Eisner):

Stationary non-uniform flow without considering the streamline-curvature

$$i - \frac{dT_m}{dx} = \frac{\lambda}{2g} \cdot \left(\frac{Wm^2}{\frac{F}{U}} \right) + \frac{d}{dx} \left(v \cdot \frac{Wm^2}{2g} \right)$$

$$\text{Umformung: } S_0 - \frac{dh_m}{dx} = \frac{\lambda}{2g} \cdot \left(\frac{Vm^2}{\frac{A}{U}} \right) + \frac{d}{dx} \left(\alpha \cdot \frac{Vm^2}{2g} \right)$$

$i = S_0$ Bottom slope of the river

$T_m = h_m$ average depth

X flow direction

λ roughness coefficient

μ Manning's coefficient

F = A area

U wetted perimeter

r_h hydraulic radius

$W_m = V_m$ average velocity

g acceleration due to gravity

$v = \alpha$ Speed, altitude compensation value > 1

r Radius

w radial velocity

αZ centripetal

$\chi = \zeta^2 \zeta'$ constant

The index 0 denotes the values for uniform outflow

3. Basic assumptions:

If at any point of an otherwise straight prismatic bed an obstacle is installed (a weir, a lock or the like) or the regular limitation otherwise permit is interrupted, so may be the flow in the channel not always uniform. It will set a stationary state is non-uniform motion, which depends both on the type of disorder as on the nature of the bed.

“Elements of Technical Hydromechanics “ By Dr. Richard Von Mises

4. Methodology:

Based on the Bernoulli equation, for stationary flow of an frictionless fluid in a flow tube, the hydraulic energy level is measured from a freely selectable reference horizon which composed of geodetic height (z) as an expression of potential energy, pressure height hD ($= p / \rho g$) as an expression of the pressure energy and velocity head ($v^2 / 2g$) as an expression the kinetic energy.

$$hE = z + p / \rho g + v^2 / 2g$$

hE - energy level

h_p - The piezometer height or potential energy is the sum of geodetic height and pressure altitude

$$h_p = z + p / \rho g$$

for straight flows with a free liquid surface water level line and piezometer line are identical.

The individual components of the energy levels (energy position, altitude, speed, altitude, loss of height) are plotted as lines in an appropriate form of reference horizon (BH) from the top.

4.1. Loss of hydraulic energy:

When stationary - uniform flow will not change with time, i.e. Base slope, water surface slope and energy slope are parallel to each other and loss of energy is about the same size range.

$$z_1 + p_1/\rho g + v_1^2 / 2g = z_2 + p_2/\rho g + v_2^2 / 2g + h_v$$

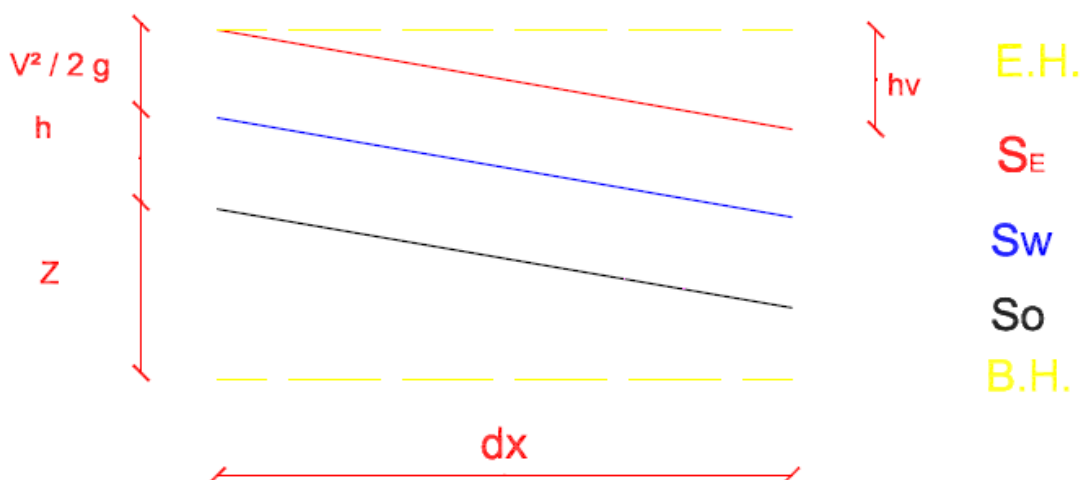


Figure 1: Stationary non-uniform flow

So = bottom slope
 SE = energy gradient
 WS = water level line
 SW = SE = So
 hv hydraulic losses
 E.H energy horizon
 B.H reference horizon
 dx distance between 1-2

5. Differential equation of the water level line in stationary non-uniform flow:
 5.1. without consideration of the current filaments (Fig. 2):

Following the change of slope, roughness and cross section in the flow direction change velocity and depth between cross sections 1 and 2.

$$Z_1 + h_1 + \frac{v_1^2}{2g} = Z_2 + h_2 + \frac{v_2^2}{2g} + h_v \quad 1-2 \text{ Excluding the streamline curvature of the distance}$$

$$dhp = dh + dz = -d\left(\frac{v^2}{2g}\right) - dh_v$$

$$dhp = -S_E \cdot dx - d\left(\frac{v^2}{2g}\right) \quad \text{oder} \quad dh = (S_o - S_E) dx - d\left(\frac{v^2}{2g}\right)$$

$$\text{It follows: } \frac{dh}{dx} = (S_o - S_E) - \frac{d}{dx}\left(\frac{v^2}{2g}\right)$$

With the resistance force from stationary-uniform flow $\frac{\lambda}{2g} \cdot \left(\frac{v^2}{A}\right)$ and taking into account the velocity distribution of the velocity or height compensation coefficient (α) in a cross section results in the differential equation of the water level line:

$$S_o - \frac{dh}{dx} = \frac{\lambda}{2g} \cdot \left(\frac{v^2}{A}\right) + \frac{d}{dx}\left(\alpha \cdot \frac{v^2}{2g}\right)$$

- 5.2. Energy balance values:
 5.2.1. Speed, altitude compensation value "α":

For currents in open or closed conduits under normal conditions, the kinetic energy is greater in a cross-section of 8.5 to 15% than the calculated value with the average velocity i.e. if the velocity head or kinetic energy with the average velocity is formed, so this value ($v^2 / 2g$) with the coefficient α are multiplied ($\alpha \cdot v^2 / 2g$) and thus the influence of the non-uniform velocity distribution in the flow cross-section on the kinetic energy to be considered.

$$\alpha \approx 1.08 - 1.15$$

- 5.2.2. "ζ" Pressure altitude compensation value:

In dem Fall, dass die Piezometer-Höhe nicht in allen Punkten eines Fließquerschnitts gleich ist, ist eine weitere Korrektur der Energiegleichungen erforderlich. Das kommt vor wenn, sich die Flüssigkeit auf gekrümmten Bahnen bewegt, z.B. beim Ausfluss unter einer Schütze oder Überfall über ein Wehr.

In der gekrümmten Strömung wird die Piezometerhöhe (h_p) mit dem Druckhöhenausgleichwert multipliziert.

ζ' bezeichnet den Anteil der wirksamen Höhe bzw. potentiellen Energie für die gekrümmte Strömung an jenem Querschnitt.

Aus diesen zwei Beiwerten ergibt sich der Beiwert $\chi = \zeta' * \zeta \approx 1/\beta$ (aus Untersuchungen von Boussinesq).

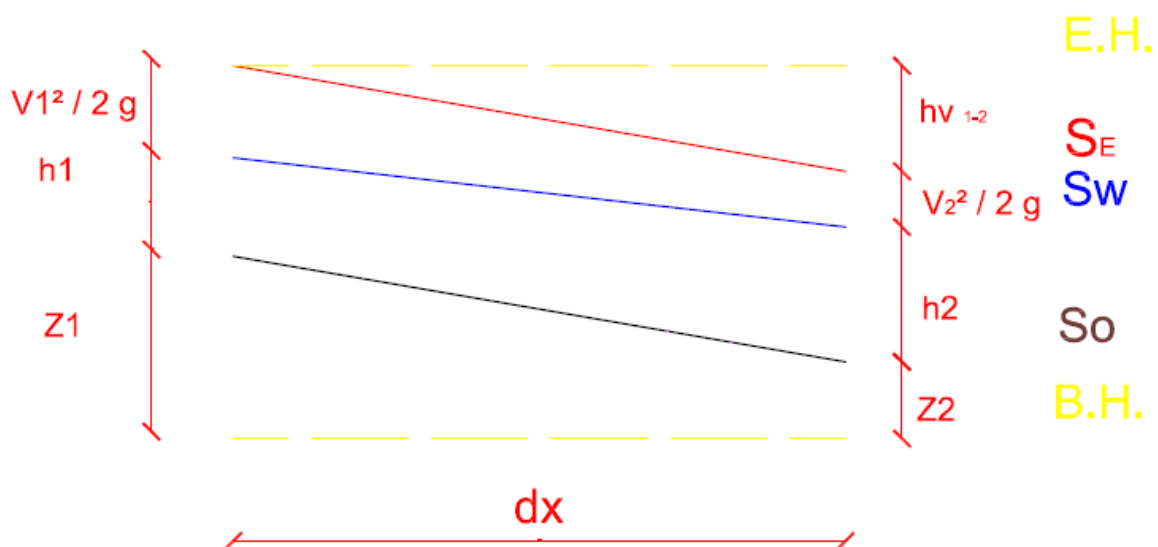


Figure 2: Stationary non-uniform flow - excluding the streamline curvature

$$dz = z_2 - z_1, \quad dh = h_2 - h_1, \quad dh_v = h_{v2} - h_{v1}$$

$$dh_p = dh + dz$$

$$S_o = dz/dx, \text{ Sohlengefälle}$$

$$S_E = dh_v/dx \text{ Energiegefälle}$$

$$d(v^2/2g) = v_2^2/2g - v_1^2/2g$$

dx Abstand zwischen 1 und 2

SW Wasserspiegellinie

E.H Energiehorizont

B.H Bezugshorizont

5.3. Taking into account the curvature of the flow filaments $\left(\frac{d^2h}{dx^2}\right) \cdot a z :$

In rivers above the points of transition from non-uniform to uniform motion must not neglect the curvature of the mirror flow, e.g. in mountain streams in the cessation of the uniform motion by the formation of an almost sudden rising water jump.

According to "Euler" an acceleration corresponding flow in any direction, the static gradient in this direction

$$w / g = \text{gradient} (p / (\rho \cdot g) + h)$$

The change in the static height in the interior, to the extent of the be different on the surface, as long as the applied acceleration component (centripetal) in the depth direction.

The centripetal acceleration is caused by the action of the centripetal force, which is responsible for the deflection of the body by the rectilinear motion.

Centripetal force balances centrifugal force, but in the opposite direction.

$w = v / r$ radial velocity

$$a z = w^2 r$$

$$V^2 / rg = d (p / \rho g + h) / dr$$

so: $Vo = \zeta \cdot Vm$ where Vo is the speed at the surface.

$$\frac{(\zeta \cdot Vm)^2}{g} \cdot \frac{d^2h}{dx^2} \cdot (\zeta' \cdot h) \text{ corresponds to the amount of change of the static gradient on a}$$

cross section, i.e. in each cross section is the mean of the static height greater by this measure than its value at the surface.

The gradient is effective only on a fraction of the depth, therefore, a fraction of the depth is taken $(\zeta' \cdot h)$.

During a movement in the direction of flow from a point to point, these factors vary ζ, ζ' .

From studies of Boussinesq stating that: $\chi = \zeta^2 \cdot \zeta' \approx 1/3$ und $\approx 1.08 - 1.15$

$$\frac{d}{dx} \left(\chi \cdot \frac{Vm^2}{g} \cdot \frac{d^2h}{dx^2} \right)$$

5.3.1 Main formula:

$$So - \frac{dh}{dx} - \frac{d}{dx} \left(\chi \cdot \frac{Vm^2}{g} \cdot \frac{d^2h}{dx^2} \right) - \frac{\lambda}{2g} \cdot \left(\frac{Vm^2}{U} \right) = \frac{d}{dx} \left(\alpha \cdot \frac{Vm^2}{2g} \right)$$

Prior to the curvature as for the curvature (at the transition) is valid for the sizes of the index α :

$h_o \cdot y$ is the elevation of the level above the normal level at any point?

$$V = V_o,$$

$$h = h_o,$$

$$\lambda = \lambda_o,$$

$$A = A_o,$$

$$U = U_0$$

$$A = A_0 + b_0(h_0 \cdot y) = A_0(1 + y)$$

$$V = V_0 \cdot A_0/A = V_0/(1 + y) \approx V_0 \cdot (1 - y)$$

$$V^2 \rightarrow V_0^2(1 - 2y)$$

$$U = U_0 + (h_0 \cdot y)(1/\sin \theta_1 + 1/\sin \theta_2) = U_0(1 + \theta y)$$

$$\text{As a shortcut: } \theta = A_0/U_0 \cdot (1/\sin \theta_1 + 1/\sin \theta_2)$$

θ is the slope angle (for channels of great width, relatively low Deep and not too shallow slope angles θ which is very small number)

$$1/(2(A/U)) \approx 1/(2(A/U)_0) \cdot [1 + (\theta - 1)y]$$

The first link:

$$dh = h_0 \cdot dy;$$

$$d\sigma = \frac{dx}{h_0};$$

$$y' = \frac{dy}{d\sigma}$$

$$S_0 - \frac{dh}{dx} = S_0 - h_0 \frac{dy}{dx} = S_0 - \frac{dy}{d\sigma} = S_0 - y'$$

The second link:

$$\frac{d}{dx} \left(\chi \cdot \frac{V m^2}{g} \cdot \frac{d^2 h}{dx^2} \right),$$

$$\chi = h_0 \cdot \sigma$$

$$\text{If } \frac{d^2 h}{dx^2} = \frac{d^2(h_0 \cdot y)}{h_0^2 \cdot d\sigma^2} = \frac{1}{h_0} \cdot y''$$

$$\text{Then } \chi \frac{V_0^2}{g} h_0 \frac{d}{h_0 d\sigma} \left(\frac{1}{h_0} \cdot y'' \right),$$

$$\text{then for the 2nd link} = \chi \cdot \frac{V_0^2}{h_0 \cdot g} \cdot y'''$$

Third element:

$$\frac{\lambda}{2g} \cdot \left(\frac{V^2}{U} \right) = 2 \cdot \left(\frac{\lambda_0}{\left(\frac{A}{U} \right)_0} \right) \frac{V_0^2}{2g} (1 - 2y)[1 + (\theta - 1)y]$$

$$= \frac{\lambda}{\left(\frac{A}{U} \right)_0} \cdot \frac{V_0^2}{g} \cdot (1 - 2y) \cdot [1 + (\theta - 1)y]$$

$$= \frac{\lambda}{g} \cdot \frac{Vo^2}{\left(\frac{A}{U}\right)_o} \cdot [1 - (3 - \emptyset) y]$$

$$hm \approx \frac{A}{U}, \text{ und } hom = \left(\frac{A}{U}\right)_o, \lambda = \lambda_o$$

$$\text{Then for the 3rd link: } \frac{\lambda_o Vo^2}{g hom} [1 - (3 - \emptyset) y]$$

fourth element:

$$\frac{d}{dx} \left(\alpha \cdot \frac{V^2}{2g} \right); V^2 \rightarrow Vo^2 (1 - 2y)$$

$$= \alpha \cdot \frac{Vo^2}{2g} \frac{d}{dx} (1 - 2y);$$

$$dx = d\sigma \cdot ho;$$

$$y' = \frac{dy}{d\sigma}$$

$$\text{Then for the 4th link: } = -\alpha \cdot \frac{Vo^2}{ho \cdot g} y'$$

From the 1,2,3 and 4 gives:

$$So - y' - \chi \cdot Vo^2 / (ho \cdot g) \cdot y'''' - \lambda_o / g \cdot vo^2 / hom [1 - (3 - \emptyset) y] + \alpha \cdot (Vo^2) / (ho \cdot g) y = 0$$

$$So - y' - \chi \cdot \frac{Vo^2}{ho \cdot g} \cdot y'''' - \frac{\lambda_o Vo^2}{g hom} + (3 - \emptyset) y \cdot \frac{\lambda_o Vo^2}{g hom} + \alpha \cdot \frac{Vo^2}{ho \cdot g} y' = 0$$

$$So = \frac{\lambda_o Vo^2}{g hom}$$

$$\frac{\lambda_o Vo^2}{g hom} - y' - \chi \cdot \frac{Vo^2}{ho \cdot g} \cdot y'''' - \frac{\lambda_o Vo^2}{g hom} + (3 - \emptyset) y \cdot \frac{\lambda_o Vo^2}{g hom} + \alpha \cdot \frac{Vo^2}{ho \cdot g} y' = 0$$

$$y' - \chi \cdot \frac{Vo^2}{ho \cdot g} \cdot y'''' + (3 - \emptyset) y \cdot \frac{\lambda_o Vo^2}{g hom} + \alpha \cdot \frac{Vo^2}{ho \cdot g} y' = 0$$

$$\text{divide by } \left(-\chi \cdot \frac{Vo^2}{ho \cdot g} \right)$$

$$y' \cdot \frac{ho \cdot g}{\chi \cdot Vo^2} + y'''' - (3 - \emptyset) y \cdot \frac{\lambda_o ho}{\chi hom} - \frac{\alpha}{\chi} y' = 0$$

$$S_o = \frac{\lambda_o V_o^2}{g h_o m} ; \frac{V_o^2}{g} = \frac{S_o \cdot h_o m}{\lambda_o}$$

then:

- Formula for transition area for each cross-section:

$$y'''' + \left(\frac{y'}{\chi}\right) \left[\frac{h_o}{h_o m} \cdot \frac{\lambda_o}{S_o} - \alpha\right] - \frac{y}{\chi} \cdot \lambda_o (3 - \emptyset) \cdot \frac{h_o}{h_o m} = 0$$

\emptyset is neglected

For very ready channel $h_o m \approx 2h_o$

$$y'''' + \left(\frac{y'}{2\chi}\right) \cdot \left(\frac{\lambda_o}{S_o} - 2\alpha\right) - \frac{3\lambda_o}{2\chi} \cdot y = 0$$

Applies to rivers and streams

$$y'''' + p y' = q y$$

5.3.2. Solving the differential equation :

p and q are constant,

$$p = \frac{1}{2\chi} \left(\frac{\lambda_o}{S_o} - 2\alpha\right) q = \frac{3\lambda_o}{2\chi}$$

$$y = A_1 \cdot e^{x_1\sigma} + A_2 \cdot e^{x_2\sigma} + A_3 \cdot e^{x_3\sigma}$$

A1, A2, A3, the three integration constants and x1, x2, x3, the three roots

$$x^3 + px = q \text{ Reduced cubic equation}$$

6. Stream change:

Following the change of slope, the wall roughness and the geometry of the cross section in an open channel causes a flow change.

By the above changes; the flow goes from one state to another for e.g. currents for shooting as an obstacle or vice versa from shooting to flow as an extension of the channel.

Due to the Froude number, we can see what the present flow condition is:

Fr. = stream velocity / wave velocity

Fr. < 1 Streaming

Fr. > 1 shooting

Fr. = 1 Critical Condition

There are two types of flow changing:

Accelerating the transition from the streams for the shooting

Delay in the transition from shooting to streaming

In the transition from shooting to streaming, there are three ways to hydraulic jump:

On the basis of the critical depth, and the conjugate depths h_1 and h_2 is determined what type of hydraulic jump takes place. Therefore, the following formulas for stationary non-uniform flow used:

$$h_{crit.} = \sqrt[3]{\frac{Q^2}{gb^2}}$$

where Q is the runoff, the channel width b , and g is the gravitational acceleration.

$$h_1 = \frac{h_2}{2} (\sqrt{1 + 8Fr^2} - 1)$$

$$h_2 = \frac{h_1}{2} (\sqrt{1 + 8Fr^2} - 1)$$

h_1 is the conjugate water depth before the hydraulic jump and h_2 is the conjugate depth for the hydraulic jump

6.1. Case 1: top roller (to aspiring state)

This case is the optimum condition to convert energy by a jump.

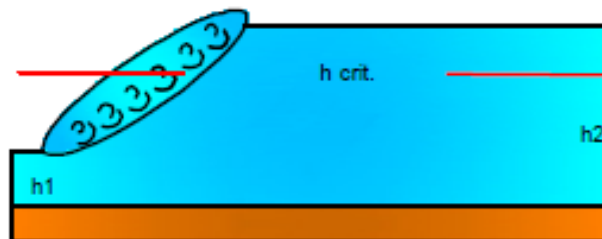


Figure 3: top roll conditions:

$Fr. > 2.2$,

$$\left(\frac{h_1}{h_{crit.}}\right) < 0.61,$$

$$\left(\frac{h_2}{h_1}\right) > 2.51$$

6.2. Case 2 hydraulic jump roller with a small deck and standing waves (critical condition):

This is caused by improper sizing of the sole of a small jump, followed by waves.

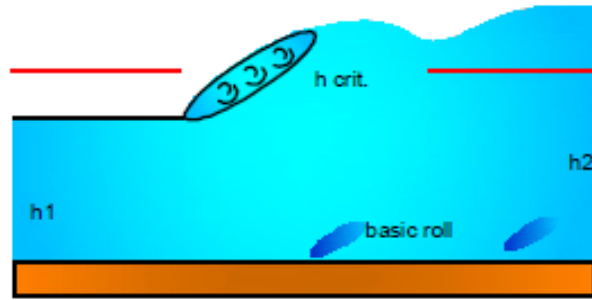


Figure 4: Hydraulic Jump with a small roller cover and standing waves conditions:

$$1.82 < Fr. < 2.2 ,$$

$$0.61 < \left(\frac{h1}{h_{crit.}} \right) < 0.67,$$

$$2.13 < \left(\frac{h2}{h1} \right) < 2.51$$

2.14

6.3. Case 3: standing waves

In this case, the hydraulic jump occurs in the form of standing waves

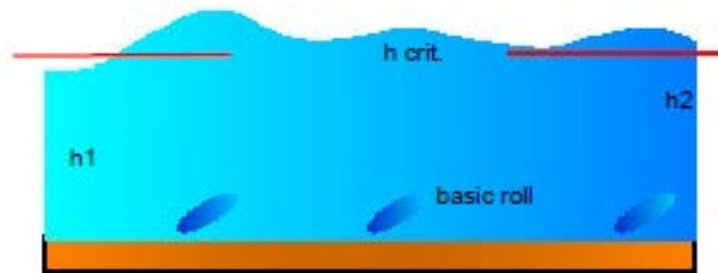
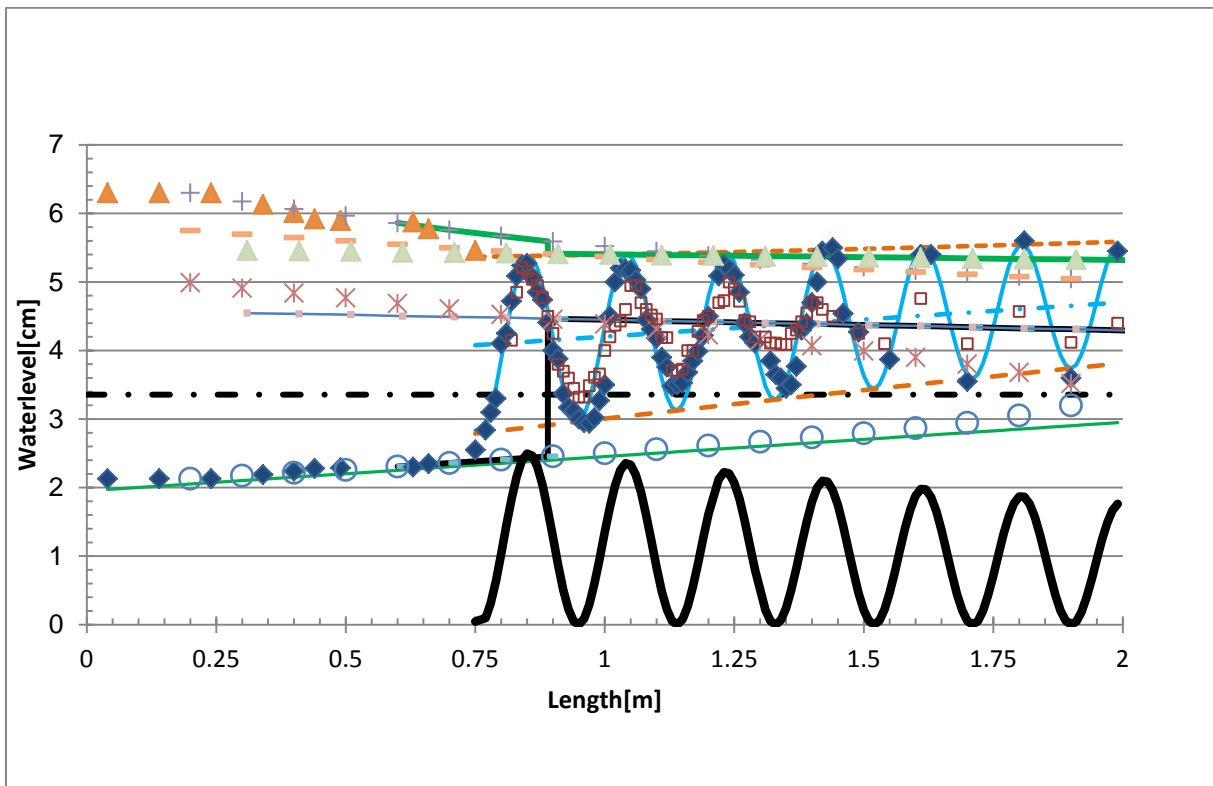


Figure 5: Standing waves conditions:

$$Fr. < 1.82 , \left(\frac{h1}{h_{crit.}} \right) > 0.67, \left(\frac{h2}{h1} \right) < 2.1$$

7. Calculation:

A 0.3 m wide Glass flume with a discharge of 0.00578 cubic meter pro second and a Manning coefficient of $107 \text{ m}^{(1/3)} / \text{s}$:



Dobberet 107 / Diagram

7.1. Calculations of the water level from the diagram:

At $x = 0.89 \text{ m}$

7.1. Calculations of the water level from the diagram:

at $x = 0.89 \text{ m}$

is there a hydraulic jump on the case-3 (ie standing waves) give

$$\text{Fr. } 1 = 1.59 > 1.82, h_1/h_{\text{krit.}} = 0.76 > 0.67, h_2/h_1 = 0.56 > 2.13$$

Table 1: Darting state

Length m	Depth cm	Depth m (h1)	V m/s	V ² /2g m	H.E. m	Fr.1	Fr.1 ²	h2 m	Fr.2
0,04	2,130	0,0213	0,9045383412	0,0417	0,06300182	1,979	3,9157	0,049901	0,5518
0,14	2,130	0,0213	0,9045383412	0,0417	0,06300182	1,979	3,9157	0,049901	0,5518
0,24	2,130	0,0213	0,9045383412	0,0417	0,06300182	1,979	3,9157	0,049901	0,5518
0,34	2,200	0,0220	0,8757575758	0,0391	0,06109028	1,885	3,5537	0,048674	0,5728
0,40	2,240	0,0224	0,8601190476	0,0377	0,06010667	1,835	3,3667	0,047994	0,5850
0,44	2,280	0,0228	0,8450292398	0,0364	0,05919523	1,787	3,1926	0,047330	0,5974
0,49	2,290	0,0229	0,8413391557	0,0361	0,05897806	1,775	3,1509	0,047166	0,6005
0,63	2,310	0,0231	0,8340548341	0,0355	0,05855604	1,752	3,0698	0,046841	0,6068
0,66	2,350	0,0235	0,8198581560	0,0343	0,05775930	1,708	2,9157	0,046202	0,6194
0,75	2,400	0,0240	0,8027777778	0,0328	0,05684670	1,654	2,7372	0,045422	0,6354
0,80	2,420	0,0242	0,7961432507	0,0323	0,05650602	1,634	2,6699	0,045116	0,6419
0,85	2,440	0,0244	0,7896174863	0,0318	0,05617858	1,614	2,6048	0,044813	0,6484
0,89	2,4550	0,024550	0,7847929396	0,0314	0,05594144	1,59917	2,5573	0,044587	0,6534

Length m	Tiefe cm	Depth m (h1)	v m/s	v ² /2g m	H.E. M	Fr.1	Fr.1 ²	h2 m	Fr.2
1,99	4,230	0,0423	0,4554767533	0,0105738569	0,0529	0,707	0,4999	0,0261	1,4554
1,9	4,250	0,0425	0,4533333333	0,0104745724	0,0530	0,702	0,4929	0,0260	1,4676
1,81	4,310	0,0431	0,4470224285	0,0101849669	0,0533	0,687	0,4726	0,0256	1,5044
1,7	4,380	0,0438	0,4398782344	0,0098620215	0,0537	0,671	0,4503	0,0251	1,5484
1,61	4,390	0,0439	0,4388762339	0,0098171432	0,0537	0,669	0,4473	0,0250	1,5548
1,5	4,400	0,0440	0,4378787879	0,0097725705	0,0538	0,666	0,4442	0,0249	1,5612
1,46	4,410	0,0441	0,4368858655	0,0097283007	0,0538	0,664	0,4412	0,0249	1,5676
1,4	4,420	0,0442	0,4358974359	0,0096843310	0,0539	0,662	0,4382	0,0248	1,5740
1,25	4,440	0,0444	0,4339339339	0,0095972813	0,0540	0,658	0,4323	0,0247	1,5870
1,1	4,450	0,0445	0,4329588015	0,0095541959	0,0541	0,655	0,4294	0,0246	1,5935
1,05	4,460	0,0446	0,4319880419	0,0095114000	0,0541	0,653	0,4265	0,0245	1,6000
1	4,490	0,0449	0,4291017075	0,0093847235	0,0543	0,647	0,4180	0,0243	1,6197
0,95	4,500	0,0450	0,4281481481	0,0093430600	0,0543	0,644	0,4152	0,0243	1,6263
0,92	4,510	0,0451	0,4271988174	0,0093016733	0,0544	0,642	0,4125	0,0242	1,6329
0,89	4,520	0,0451	0,4262536873	0,0092605610	0,0545	0,640	0,4098	0,0241	1,6396

Table 2: Pouring state

7.2. Calculation of the constants:

At the point of changing the jump $x = 0.89 \text{ m}$ gilt

$$\mu = 107 \text{ m}^{1/3}/\text{sec}$$

$$\text{Area} = 0,044587 * 0,3 = 0,01337 \text{ m}^2$$

$$\text{wetted perimeter} = 2 * 0,044587 + 0,3 = 0,3891 \text{ m}$$

$$rh = A/U = 0,01337/0,3891 = 0,034354 \text{ m}$$

$$dh = 4 * rh = 4 * 0,034354 = 0,137416 \text{ m}$$

$$\lambda = \frac{124,58}{\mu^2 \sqrt{dh}} = \frac{124,58}{11449 * 0,516} = 0,021 \text{ (Technical Hydromechanics 1 Bollrich)}$$

$$\lambda_o = \lambda = 0,021$$

$$S_o = 0.0008119 \text{ Streaming}$$

$$p = \frac{1}{2\chi} \left(\frac{\lambda_o}{S_o} - 2 \alpha \right) = \frac{1}{2 * (1/3)} * \left(\frac{0,021}{0,0008119} - 2 * 1,08 \right) = 35,7$$

$$q = \frac{3\lambda_o}{2\chi} = \frac{3 * 0,021}{2 * (\frac{1}{3})} = 0,0945$$

X1, X2, X3, the three roots of $x^3 + px = q$

Solution of the reduced cubic equation by Cardano's formula:

Discriminate $D > 0$, i.e. there is a real and two complex conjugate numbers:

$$X_1 = 0,00917$$

$$X_2 = -0,00458 + i * 3,2172$$

$$X_3 = -0,00458 - i * 3,2172$$

$$y = A_1 * e^{x_1 \sigma} + A_2 * e^{x_2 \sigma} + A_3 * e^{x_3 \sigma}$$

Can be transformed as follows:

$$y = A_1 * \exp(x_1 * x/h_o) + A_2 * \exp(x_2 * x/h_o) + A_3 * \exp(x_3 * x/h_o)$$

$$y = A_1 * \exp(x_1 * x/h_o) + A_2 * \exp((x_2 \text{ Real} + i * x_2 \text{ Imaginär}) * x/h_o) + A_3 * \exp((x_3 \text{ Real} + i * x_3 \text{ Imaginär}) * x/h_o)$$

transformed (3 constants are determined!)

$$y = A_1 * \exp(x_1 * x/h_o) + A_2 * \exp((x_2 \text{ Real} + i * A_3 * x_2 \text{ Imaginär}) * x/h_o)$$

$$+ A_2 * \exp((x_3 \text{ Real} + i * A_3 * x_3 \text{ Imaginär}) * x/h_0)$$

$$\exp(z * i) = \cos(z) + i * \sin(z)$$

$$\exp(-z * i) = \cos(z) - i * \sin(z)$$

$$\exp(z * i) + \exp(-z * i) = 2 * \cos(z)$$

$$y = A_1 * \exp(x_1 * x/h_0) +$$

$$A_2 * \exp((x_2 \text{ Real}) * x/h_0) * 2 * \cos(A_3 * x_2 \text{ Imaginär} * x/h_0)$$

$$y = A_1 * \exp(0,00917 * x/h_0)$$

$$+ A_2 * \exp((-0,00458) * x/h_0) * 2 * \cos(A_3 * (3,2172) * x/h_0)$$

assumptions:

$$A_1 = h_0 = h_2 = 0.044587 \text{ m} \quad \text{Depth in the flowing state}$$

$$A_2 = \frac{1}{4} (h_2 - h_1) = 0.0050 \text{ m} \quad \text{Quarters of the jump height}$$

$$A_3 = 0,5 \quad \text{Correction of the wavelength}$$

$$s = x/h_0 = 0.0 / 0.044587 = 0$$

$$y(0.89) = 0.044587 * e^{0.009 * 0.0} + 0.005 * e^{-0.005 * 0.0} * 2 * \cos(0.50.53.217 * 0.0)$$

$$= 0.0055534382 + 0.004435666$$

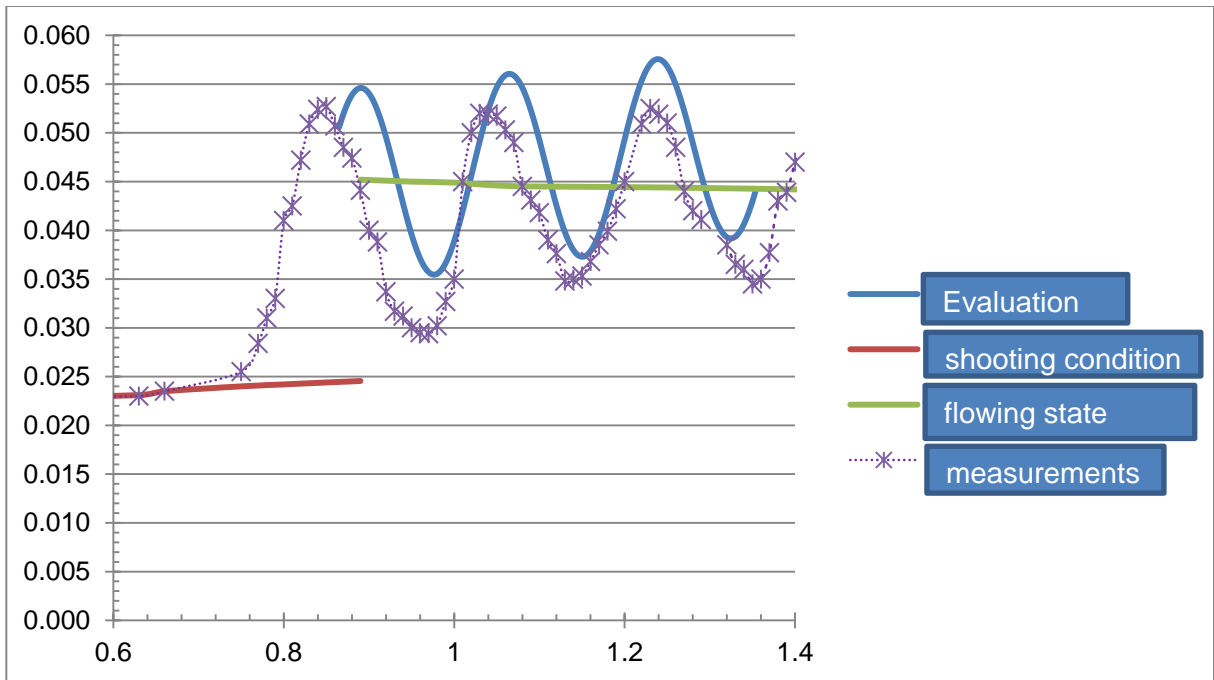
$$= 0.044587 * 1 + 0.005 * 1 * 2 * 1$$

$$= 0.054587 \text{ m}$$

Presented in such a proper evaluation of results as in the following graphic.

Height of the wave at the point of the alternating jump is 5.45 cm (measured)

7.3 Graphical representations of the wave height:



With the very simple provisions of the following three constants, it is possible to calculate the resulting wave.

A significant potential bias still exists, partly because of the definition of a bottom slope necessary, although it has been measured at a linear fall to zero.

8. Conclusion:

- The abstract solution of the differential equation of Eisner to calculate the wave height has been transformed so that the constant coefficients can be calculated from the differential equation to the experimental conditions. This is applicable to any specific attempt to back up the assumptions of the three constants.
- Due to unfavorable differences in height between the upper \rightarrow and underwater in cases 2 and 3 are formed on the surface standing waves and rollers on the sole reason. Be avoided for constructional reasons, the presence of a standing wave, as it erodes over time, the sole or the river bed, which requires a long and expensive treatment. A hydraulic jump roller with cover (as in Case 1) is desirable because there is the excess of energy in a short stretch \rightarrow converted so that the building is built economically and safely and sustained. From an ecological perspective, the erosion of the riverbed is not acceptable because it leads to the deepening of the riverbed and in turn creates a high water level and at the vortex position eroded, affecting the continuity of the water.

9. References:

1. Elements of Technical Hydromechanics of Dr. Richard von Mises,
2. Technical Hydromechanics 1, Gerhard Bollrich
3. Technical documentation Hydromechanics lecture, Prof. Jörg Kranawetterreiser
4. Handbook of Mathematics, Bronstein Semendjajew

5. Dobberet 107 / Diagram 1 (from thesis of Lisa Dobbert)