UPP 92 093

Analyses of Crack Growth Across a Bimaterial Interfaces

Bahzad Rashid

Division of Solid Mechanics, Department of Technology,
Uppsala University,
Box 534, S-751 21 Uppsala, SWEDEN

Analyses of Crack Growth Across a Bimaterial Interfaces

Bahzad Rashid

Division of Solid Mechanics, Department of Technology, Uppsala University, Box 534, S-751 21 Uppsala, SWEDEN

Abstract

Frequently materials are combined to produce some desired property, as an example include thin film semi-conductor devices, solar cells, fuel cells or optically active layers, electronic and composite laminates.

Layered structures are generally subjected to thermo-mechanical loading. This may result in initiation of cracks and growth of existing defects, eventually resulting in failure of the device. The fracture behaviour of the bimatrerial can be rationalised by stress analysis and methods of fracture mechanics.

The behaviour of cracks crossing interfaces is discussed. The system analysed consists of two dissimilar layers only. The problem of a crack growing towards an interface between two dissimilar materials is fundamental in the understanding of the behaviour of non-homogenous materials. As cracks approach an interface the stress concentration near-crack tip is intensified or reduced, depending on the different between the material properties. Thus, the crack may be attracted to, and may advance through the interface, or may be arrested before reaching the interface.

A primary investigation on a bimaterial system have been performed by using both experimental, numerical as well as analytic techniques to estimate the variation of the plane problem of crack that terminates at the interface of a bimaterial composite when loaded on its faces in a mode I way.

Linear Elastic Fracture Mechanics

The asymptotic stress field as σ_{ij} at the tip of a crack in a homogenous isotropic linear elastic material can be expressed as an asymptotic series in polar coordinates ($\mathbf{r}$, θ) centred at the crack tip

$$\sigma_{ij} = K_I r^s f_{ij}(\theta) + K_{II} r^s g_{ij}(\theta) + T\delta_{i1}\delta_{ij} + O(\sqrt{r}),$$

Where K_I and K_{II} are denoted the stress intensity factor of opening (mode I) and shearing mode (mode II) respectively. $f_{ij}\ (\theta)$ *and* $g_{ij}\ (\theta)$ are non dimensional angular functions,

δ_{ij} is the Kronecker delta and the second order term $\boldsymbol{T}$ is a constant stress acting parallel to the crack plane.

In plan stress problem the stress function satisfy the biharmonic equation can be written as:

$$f_{ij}\ (\theta) = A\ Cos(n\theta) + B\ Cos[(n\text{-}2)\theta] + C\ Sin(n\theta) + D\ Sin\ [(n\text{-}2)\theta]$$

Where *A, B, C* and *D* are constants.

The stress components as related to strain components for homogenous isotropic linear elastic material can be expressed by the constitutive equation as:

$$\sigma_{ij} = \lambda\, \epsilon_{kk}\, \delta_{ij} + 2\ G\ \epsilon_{ij}$$

Where σ_{ij} is the stress tensor and ϵ_{ij} is the strain tensor, λ and $\boldsymbol{G}$ are Lame's two coefficients which can be expressed in terms of Young's modulus $\boldsymbol{E}$ and Poisson's ratio ν as,

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad , \quad G = \frac{E}{2(1+\nu)}$$

ϵ_{kk} is the dilatation strain and δ_{ij} is the Kronecker delta.

Introduction

Any engineering structure comprise more than one elastic material, often they contain two material with different elastic constants bounded together over plane surfaces.

The engineering structure made of bounded dissimilar material frequently contain a plane boundary that is intersected at right angles by the plane of bound.

Modeling crack-tip fields, crack initiation and crack growth in bimaterial interfaces is essential for understanding failure processes in advanced materials such as composites and ceramics.

In this report a primary investigation on a bimaterial system have been performed by using both experimental and numerical as well as analytic techniques to estimate the variation of the plane problem of a crack that terminates at the interface of a bimaterial composite and loaded on its faces in a mode I way.

The bimaterial system used in this work is the PSM-1 carbonate plastic clamped with a steel which has a relativly ver large stiffness. Material I was sawed to a different depth up to the interface point. Mode I way load were applied at each crack depth. The photoelastic fringe patterns were obtain for each crack length. The difference between the maximum and the minimum principle stresses calculated analytically by using the program Mathmatica

A numerical investigations of the problem was made. Hence a two dimensional linear elastic finite element calculation by using FE-program ADINA have been performed. The difference between the maximum and the minimum principle stresses were ploted. The results of the calculations agreed well with that measured experimentally.

Experimental procedure

The photoelastic methods, which is based upon a unique property of some transparent materials was used. The general dimensions of the specimens and of the models are shown in Fig. (1).

Tests are conducted on a bimaterial system which are consists of two material clamped together. Materil properties and models dimensions are given in Table 1 below.

Table 1

material	type	length (L)	width (W)	thickness	modulus of elasticity	Poison ratio (v)	(ϕ) fringe constant
I	PSM-1	180 mm	155 mm	2.5 mm	2.5×10^9	0.38	7 KN/m
II	Steel	180 mm	25 mm	15 mm	2.1×10^{12}	0.31	

The experimental tests were run in several steps. Material I which is PSM-1 polycarbonate plastic was sawed up to the depths 135 mm, 145 mm, 150 mm, and 155 mm respectively at points P1, P2, P3 and P4 as shown in Fig. 1. At each crack length the model was loaded in a mode I way by applying a displacement of 3 mm by a wedge. A wedge is pressed on the upper faces of the specimen perpendicular to plane of the specimen.

As photoelastic data required in the vicinity of the crack-tip, there is aneed to magnify isochromatic fringe patterns. A Natrium light, polarizer, quarter-wave plate, an analyzer and CCD-camera was used to obtain an isochromatic pictures on the computer disc. The arrangement of the system is shown in Fig. 2. A fring patterns obtains at the vicinity of the crack-tip for the different crack lengths at the same load level has been shown in Fig. 3, 4 , 5, 6. While Fig. 7, 8, 9, 10 shows a fring patterns obtains for the whole geometry at points p1, p2, p3 and p4 respectively.

The plane problem of the crack that terminates at the interface of a bimaterial

---

$\sigma = K r^{s} f(\theta)$, $s = n-2$

Material constants, (material 2 includes the crack)

```
E2:=1
ny:=0.38
E1 is very large
```

Plan stress problem, using a polar coordinates system with modus I way loading:

The stress function satisfy the biharmonic equation:

```
fi2[theta_]:= (c Cos[n theta] + d Cos[(n-2) theta] +
               e Sin[n theta] + f Sin[(n-2) theta]);

v={c,d,e,f};
```

Stress components as related to the stress function:

```
sigma2t[theta_]:= n (n-1) fi2[theta]
sigma2r[theta_]:= n fi2[theta] + fi2''[theta]
tau2rt[theta_]:= -(n-1) fi2'[theta]
```

Strain component (Hooke's law)):

```
epsilon2t[theta_]:= ((1-ny^2)/E2) (sigma2t[theta] -
              (ny/(1-ny)) sigma2r[theta])

epsilon2r[theta_]:= ((1-ny^2)/E2) (sigma2r[theta] -
              (ny/(1-ny)) sigma2t[theta])

gamma2rt[theta_]:=(2 (1+ny)/E2) tau2rt[theta]
```

The boundary condition:

```
VL3:=gamma2rt[Pi/2] - (1/(n-1)) epsilon2r'[Pi/2]
VL4:=epsilon2r[Pi/2]
HL5:=sigma2t[Pi]
```

```
HL6:=tau2rt[Pi]
```

Equation system for the unknown n,c,d,e,f (5 unknown with 4 equ.)

```
ekv= Expand[{Expand[VL3],
       Expand[VL4],Expand[HL5],
       Expand[HL6]},Trig->True];
```

To find n (the stress singularity coefficent))

The matris system with the coeff. for the unknown a,b,c,d,e,f

```
Coeff=Expand[Table[Coefficient[ekv[[i]],v[[j]]],
           {i,1,4,1},{j,1,4,1}],Trig->True];
```

The determinate:

```
Det[Coeff];
```

To simplify

```
%//Expand;

Expand[%,Trig->True];

%//Expand;

                           3   2
(4761 (-2 + n) (-1 + n)    n

                        2              (-2 + n) Pi    n Pi
   (372 + 1250 n - 625 n  + 625 Cos[----------- - ----] -
                                            2          2

                (-2 + n) Pi    n Pi
    1250 n Cos[----------- - ----] +
                     2          2

          2      (-2 + n) Pi    n Pi
    625 n  Cos[----------- - ----] +
                     2          2

              (-2 + n) Pi    n Pi
    925 Cos[----------- + ----])) / 781250
                  2          2

(4761*(-2 + n)*(-1 + n)^3*n^2*
    (372 + 1250*n - 625*n^2 +
      625*Cos[((-2 + n)*Pi)/2 - (n*Pi)/2] -
      1250*n*Cos[((-2 + n)*Pi)/2 - (n*Pi)/2] +
      625*n^2*Cos[((-2 + n)*Pi)/2 - (n*Pi)/2] +
      925*Cos[((-2 + n)*Pi)/2 + (n*Pi)/2]))/781250

-0.00164902 (723.395 + 925 Cos[0.66164 Pi] +

    273.605 Cos[1. Pi])

Factor[%,Trig->True];

%//Factor;
```

```
ExpandAll[%,Trig->True]
```

$$\frac{-1204533\, n^2}{390625} + \frac{32236731\, n^3}{781250} - \frac{106060797\, n^4}{781250} + \frac{30960783\, n^5}{156250} -$$

$$\frac{114278283\, n^6}{781250} + \frac{33327\, n^7}{625} - \frac{4761\, n^8}{625} - \frac{176157\, n^2\, \mathrm{Cos}[n\, \mathrm{Pi}]}{15625} +$$

$$\frac{1233099\, n^3\, \mathrm{Cos}[n\, \mathrm{Pi}]}{31250} - \frac{1585413\, n^4\, \mathrm{Cos}[n\, \mathrm{Pi}]}{31250} +$$

$$\frac{176157\, n^5\, \mathrm{Cos}[n\, \mathrm{Pi}]}{6250} - \frac{176157\, n^6\, \mathrm{Cos}[n\, \mathrm{Pi}]}{31250}$$

```
%//Factor;

f2[n_]=%
```

$$(4761\ (-2 + n)\ (-1 + n)^3\ n^2\ (-253 + 2500\, n - 1250\, n^2 - 925\, \mathrm{Cos}[n\, \mathrm{Pi}]))\ /\ 781250$$

To plot the obtain expression

```
Plot[f2[n],{n,-3,3},Plot Range{20,-20}]
```

Det. =0 will give the no trivial soluton, that will give the singularity order (n). we are looking for ($-1 < s < 0$):

```
FindRoot[f2[n]==0,{n,1.5}]

{n -> 1.66164}
```

$\sigma=Kr^{s} f(\theta)$, s=-(2-n) , K <-> a ...

```
2-n/.%[[1]]
```

To estimate the constants (n are known, set c=1 and find d,e,f)

```
n=1.66164;
c=1;

nekv=ekv//N;

redekv={nekv[[1]]==0,nekv[[2]]==0,nekv[[3]]==0};

FindRoot[redekv,{d,6},
{e,2.4},{f,-0.1}]

{d -> -5.90984, e -> -3.75193, f -> 1.01969}
```

The plane problem of the crack that terminates at the interface of a bimaterial:

$\sigma = K r^{s} f(\theta)$, $s = n-2$

Material constants (material 2 includes the crack)

The calculated constants for the case : E1=very large, E2=1, ny=0.38 are

```
n=1.66164;

c=1;
d=-5.90984;
e=-3.75193;
f=1.01969;
```

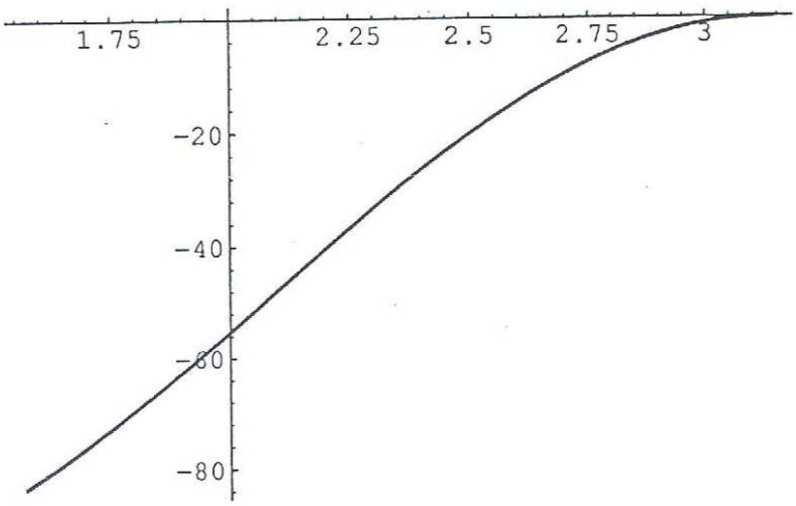
Substitute the constants to the stress function:

```
fi2[theta_]:= (c Cos[n theta] + d Cos[(n-2) theta] +
              e Sin[n theta] + f Sin[(n-2) theta]);
```

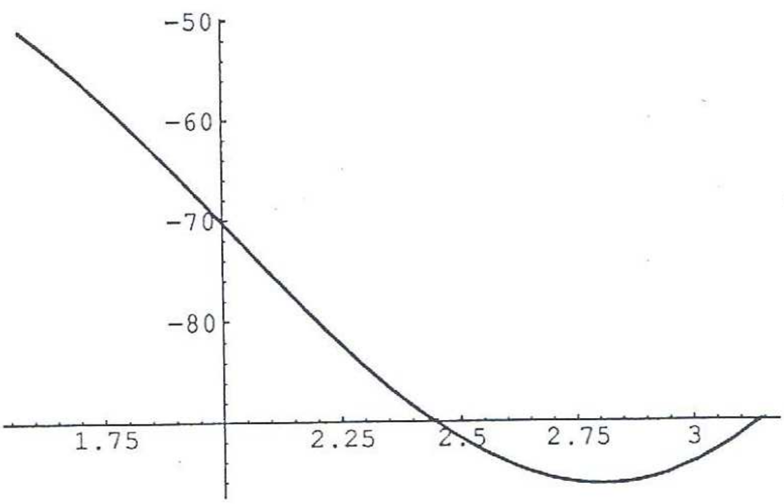
Stress components as realted to the Stress function:

```
sigt= radie^(n-2) (n (n-1) fi2[theta]);

sigr:= radie^(n-2) (n fi2[theta] + fi2''[theta]);

taurt:= radie^(n-2) (-(n-1) fi2'[theta]);

deltaprinciplestress:=2(.25(sigr-sigt)^2+taurt^2)^0.5
```

```
radie=1.5 10^-3;

p1=Plot[sigt,{theta,Pi/2,Pi}]
```

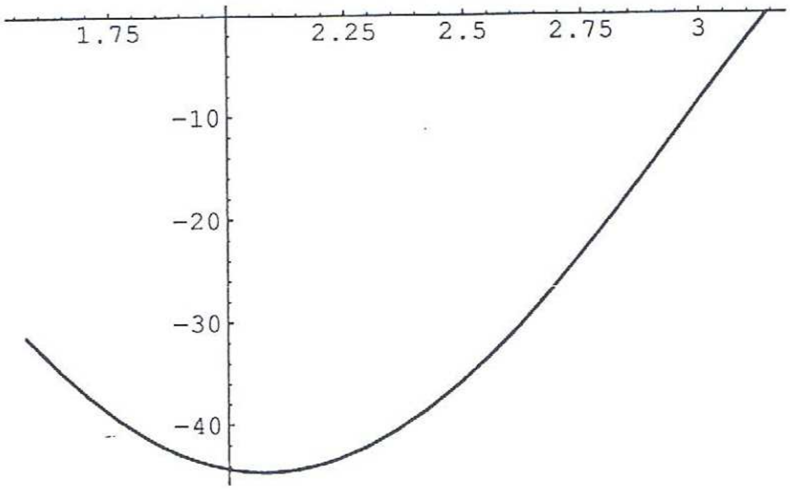



```
-Graphics-

p2=Plot[sigr,{theta,Pi/2,Pi}]
```



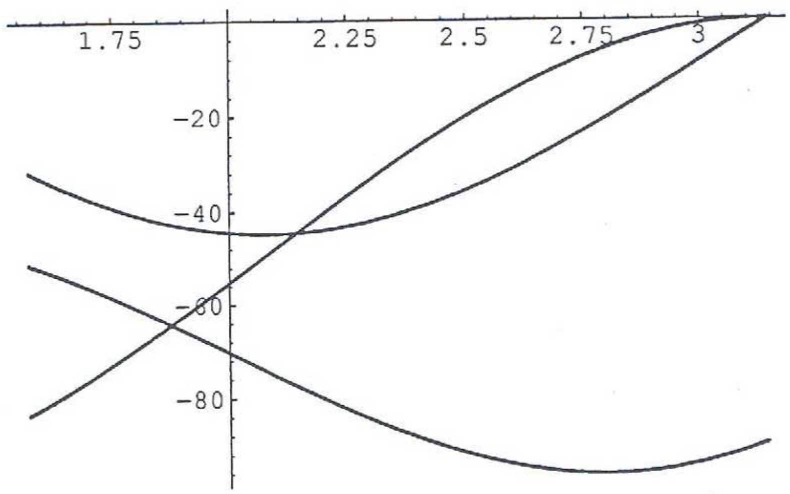

```
-Graphics-
```

```
p3=Plot[taurt,{theta,Pi/2,Pi}]
```



-Graphics-

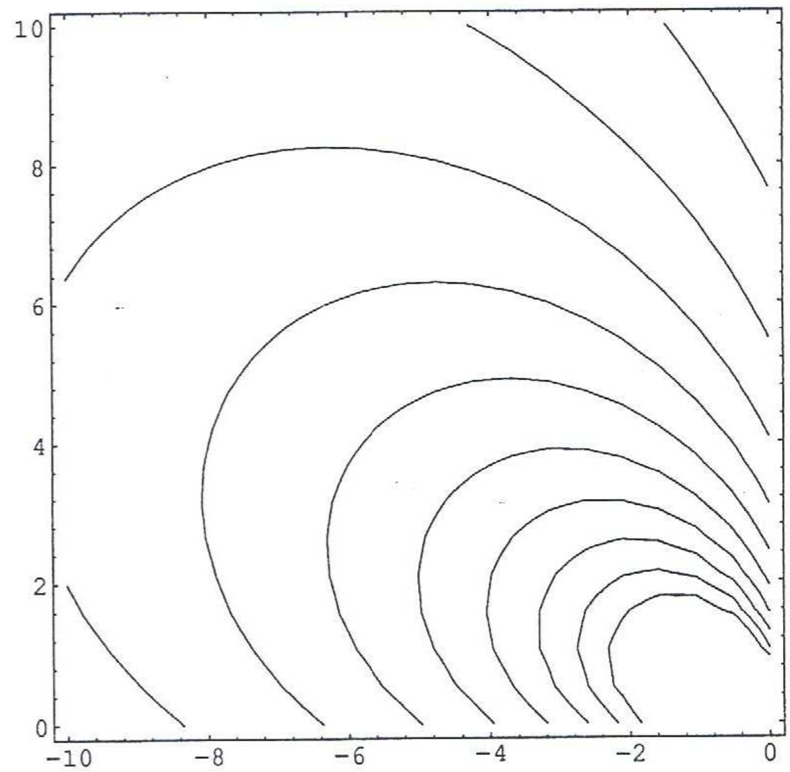
```
Show[p1,p2,p3]
```



-Graphics-

```
radie=.;

radie:=Sqrt[x^2+y^2];
theta:=ArcTan[x,y];

view1=10;
```

```
ContourPlot[2(.25(sigr-sigt)^2+taurt^2)^0.5,
          {x,-viewl,0},{y,0,viewl},

          ContourShading -> False,
          ContourSmoothing -> Automatic,
          PlotPoints->20]
```



Numerical calculation

Finite element code ADINA was used. Due to the symmetry plane of the problem, only one half of the body needs to be considered. The body was divided into 930 solid parts. There were replaced with two dimensional rectangular isoparmetric 8-nodes element. A total of 2913 nodes were used.

Fig. 11 shows the finite element mesh specified in the coordinate system y and z. The node point at C were given a displacement of 0.15mm.

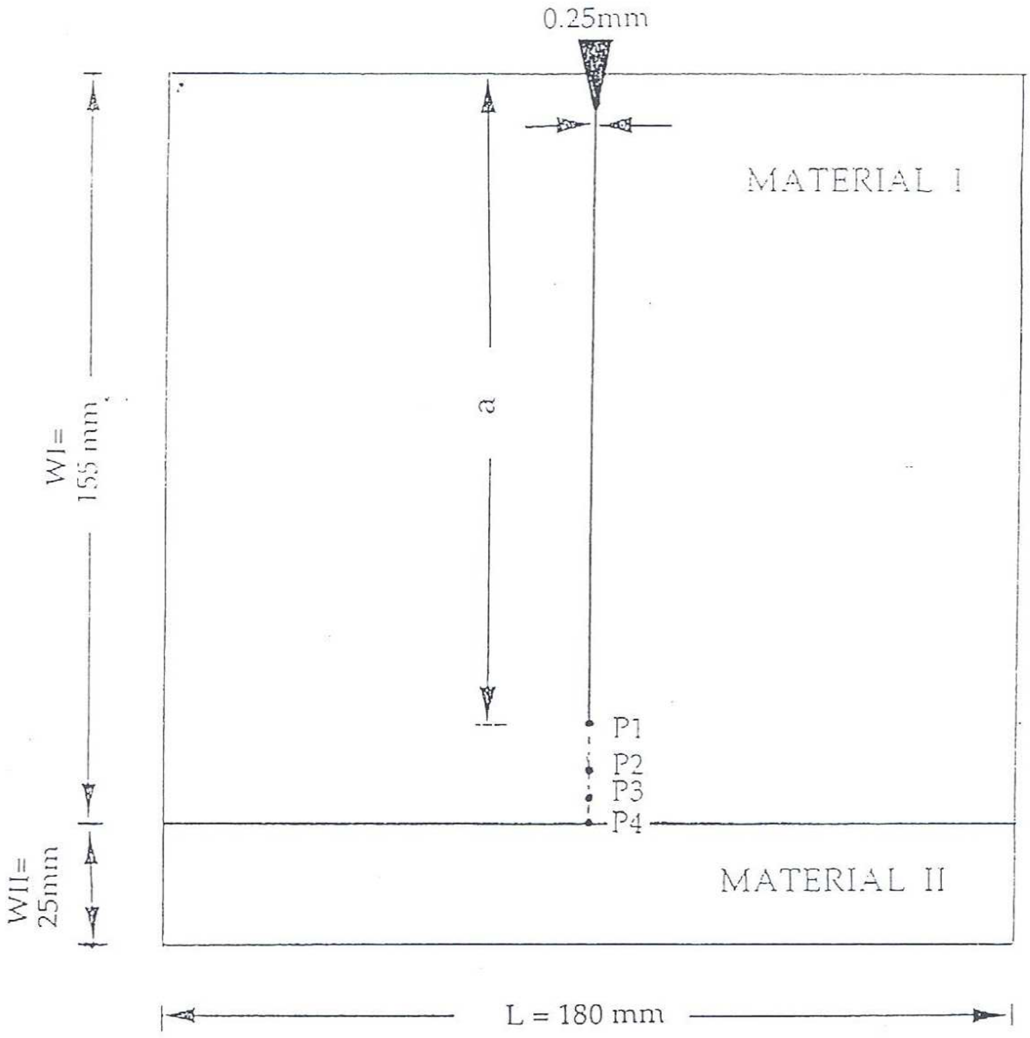
The boundary conditions applied were:

$$u_y = u_z = 0 \qquad \text{at surface AB}$$

$$\sigma_{yy} = \tau_{yz} = 0 \qquad \text{at the surface AC}$$

The principle stresses were calculated and the difference between the maximum and the minimum principle stresses were measured and ploted.

Fig. 12a, 13a, 14a, 15a shows the difference in the principle stresses for the cases were the cracks in material I reached the points p1, p2, p3 and p4 respectively. While Fig. 12b,13b,14b,15b shows the close upp of these figures.



a=135, 145, 150, 155 mm
at p1, p2, p3, p4 respectively

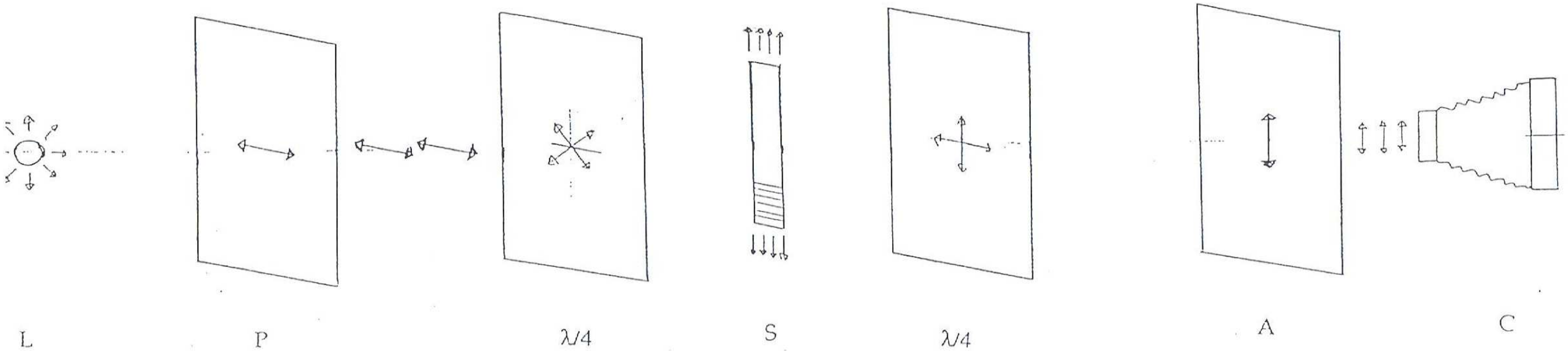
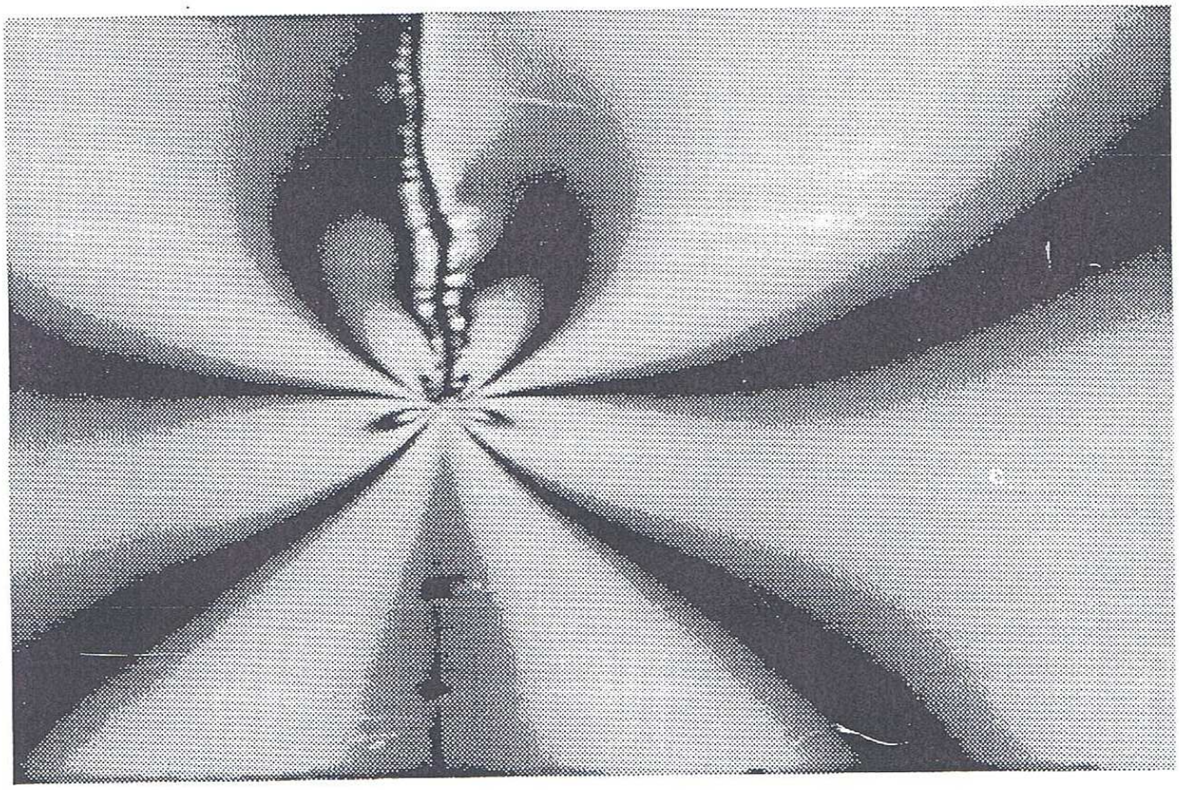


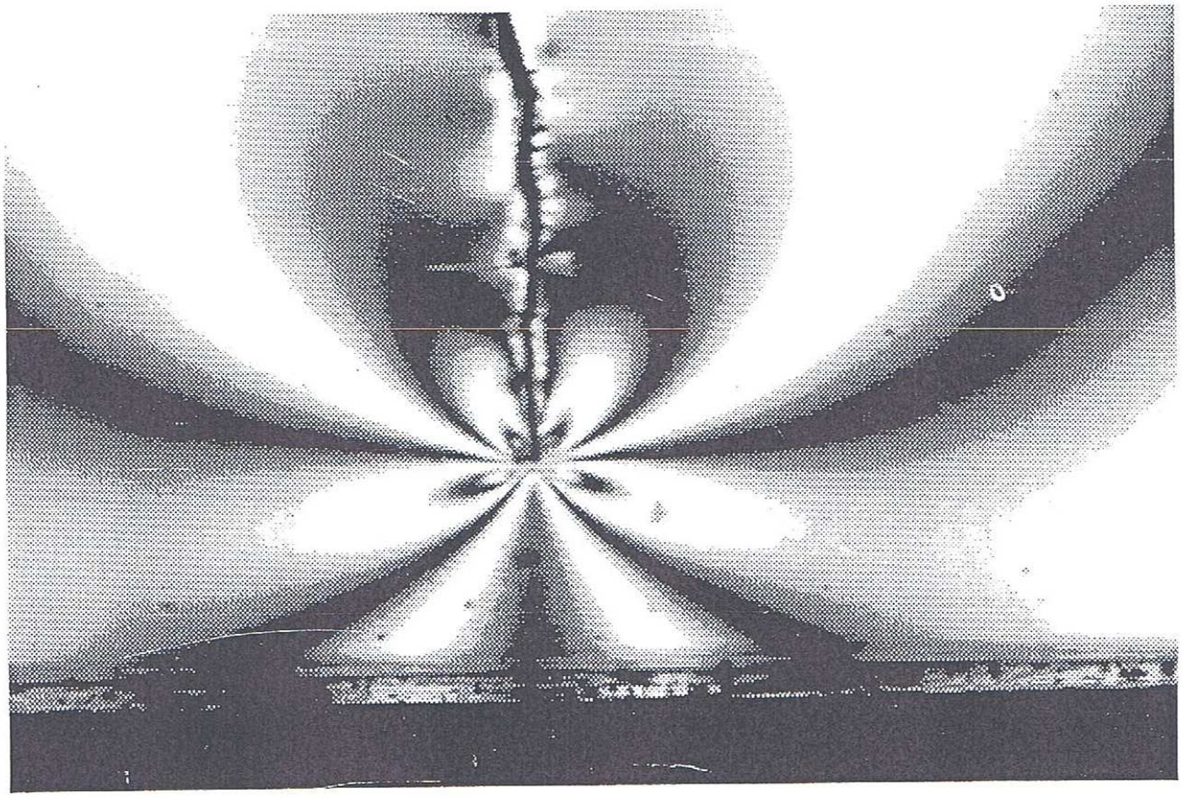
Fig. 2 Polarization of light waves in a circular polariscope.

L=Light source
P=Polarizer
λ/4=Quarter-wave plate
S=Specimen
A=Analyzer
C=Camera

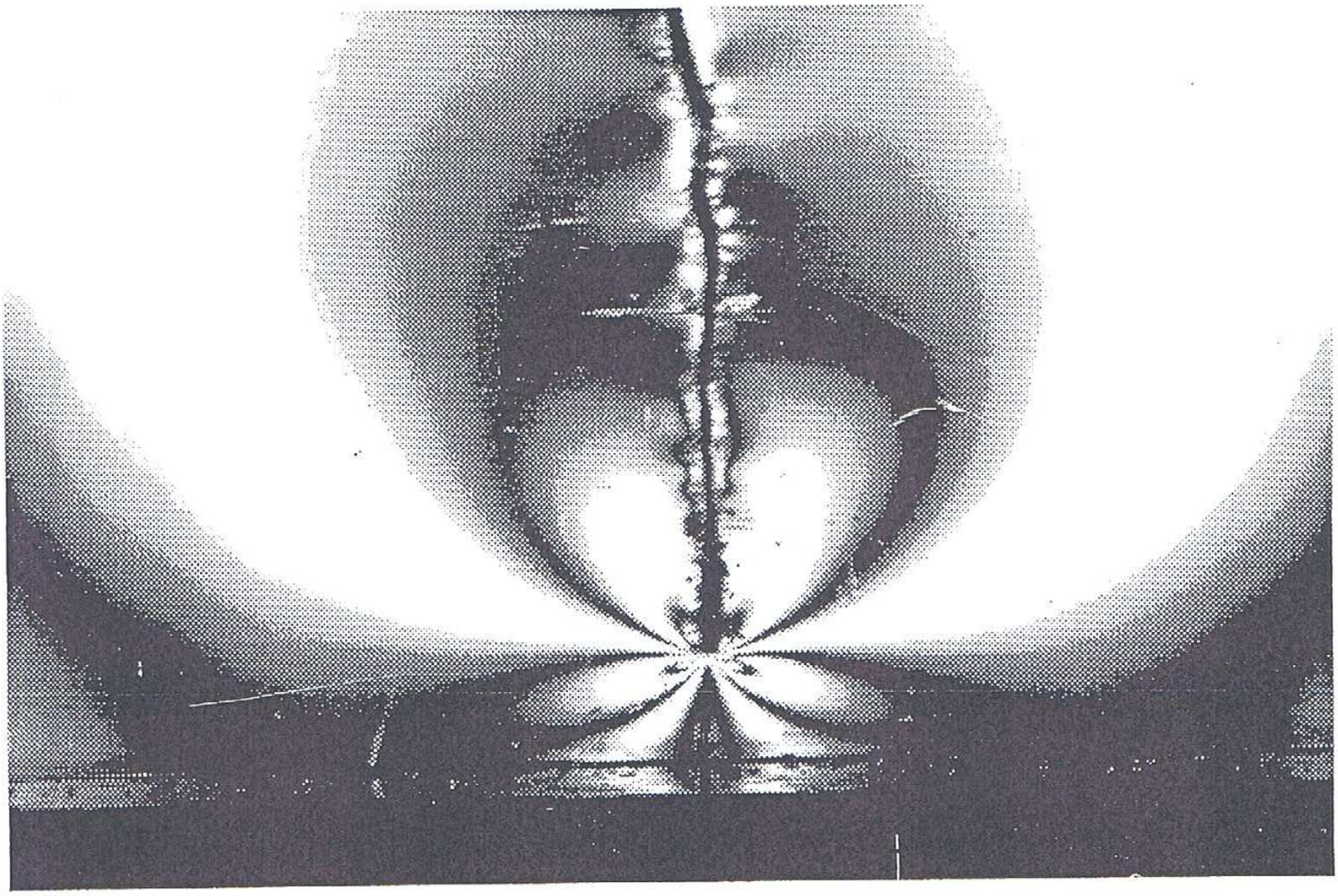
'zw-p1-2'			
Printed on:	Mon 4-Oct-1993 16:05		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0
Cells per inch:	120	Dot gain:	0%



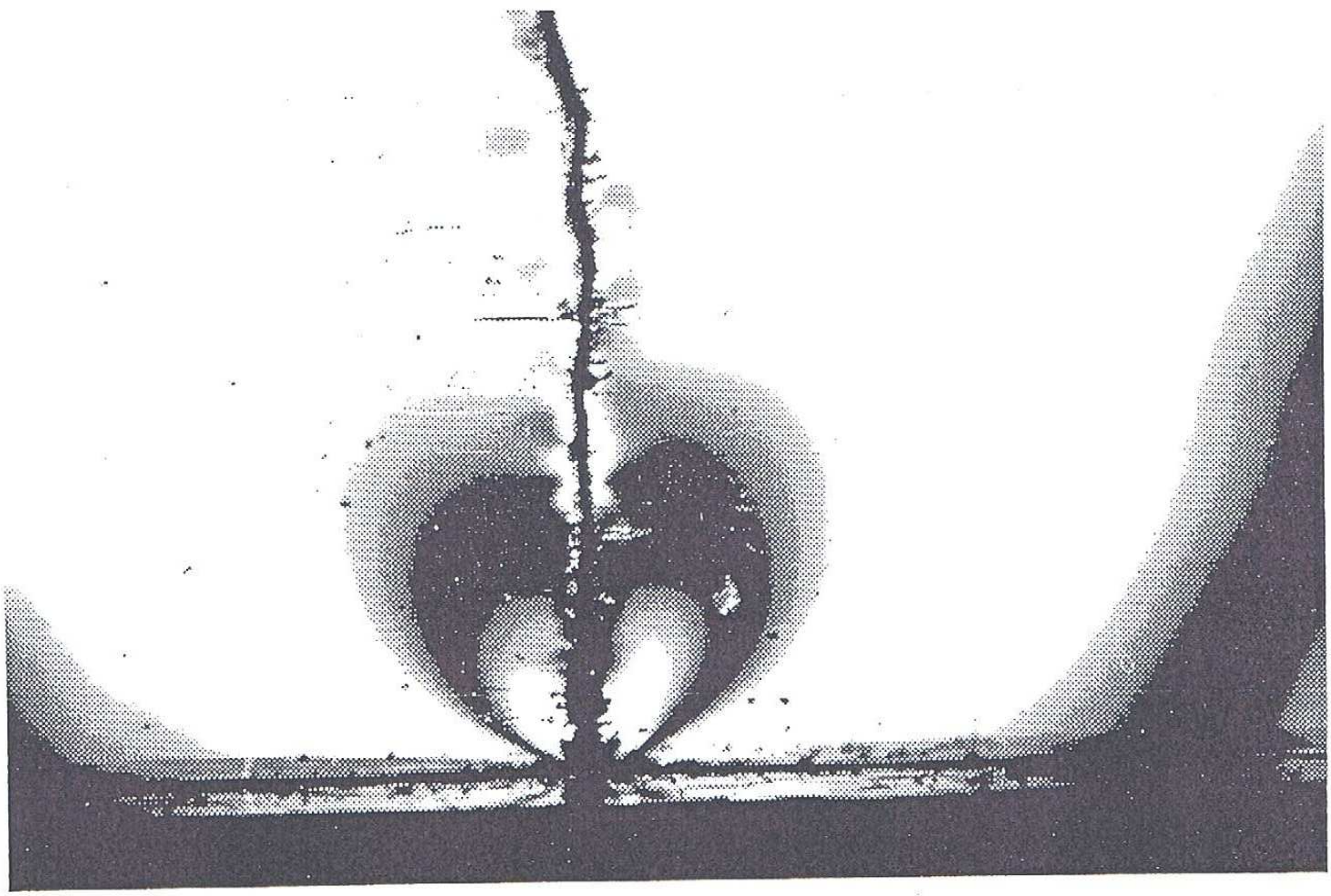
'zw-p2-2'			
Printed on:	Mon 4-Oct-1993 16:06		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0
Cells per inch:	120	Dot gain:	0%



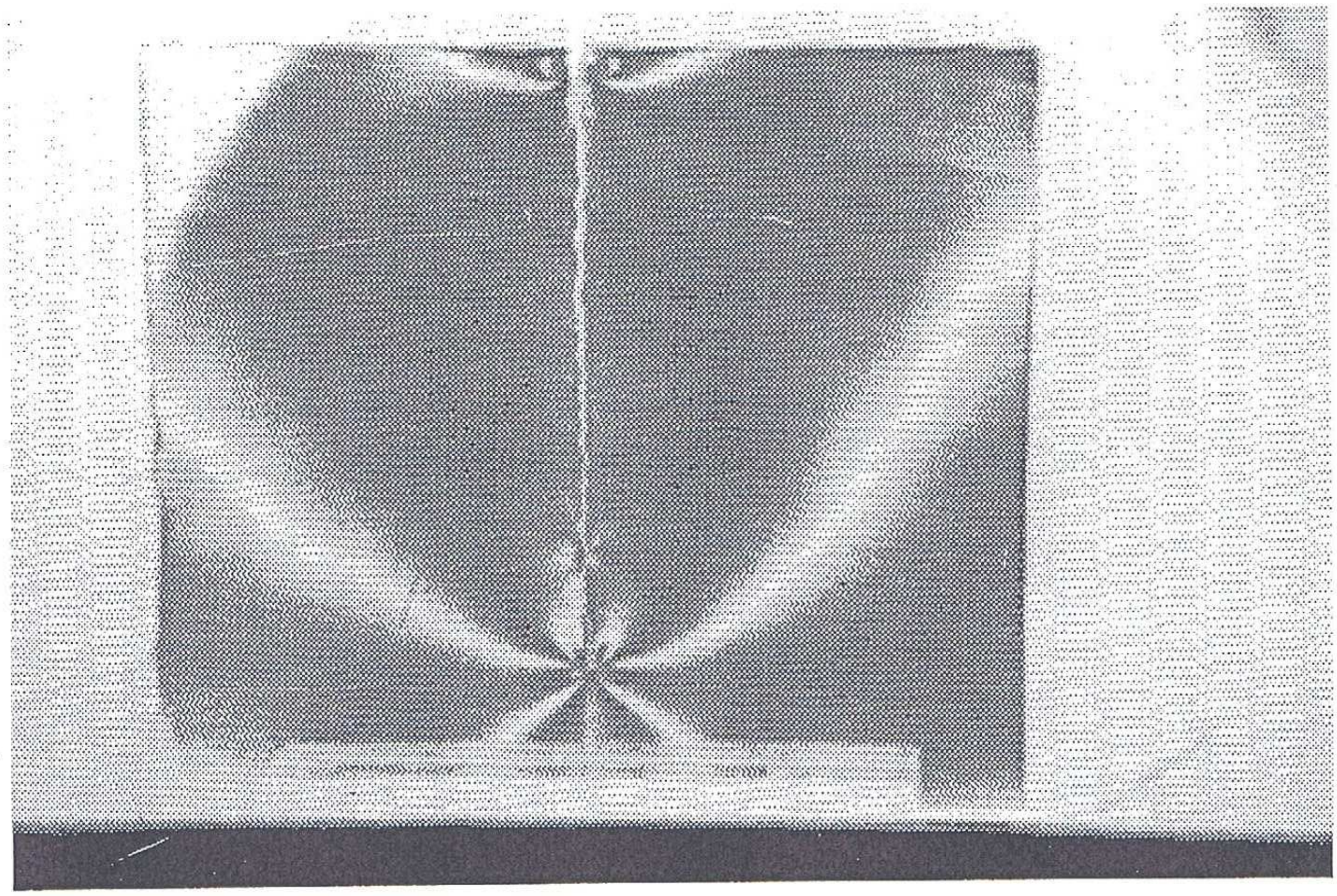
-p3-2'			
ited on:	Mon 4-Oct-1993 16:07		
een density:	Default	Screen angle:	Default
pattern:	Default	Rotation:	0
ls per inch:	120	Dot gain:	0%



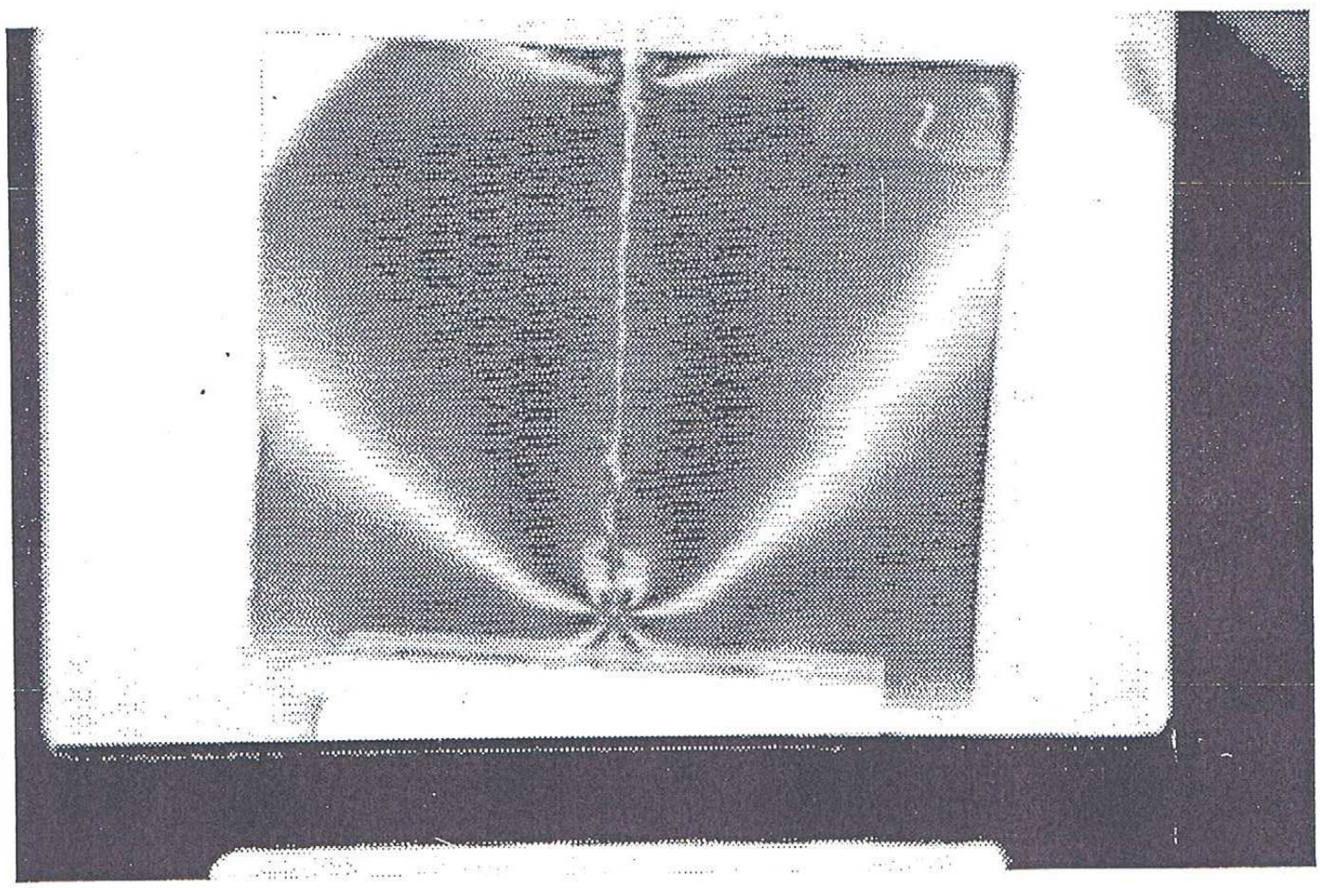
zw-p4-2'			
Printed on:	Mon 8-Nov-1993 14:34		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0˚
Cells per inch:	120	Dot gain:	0%



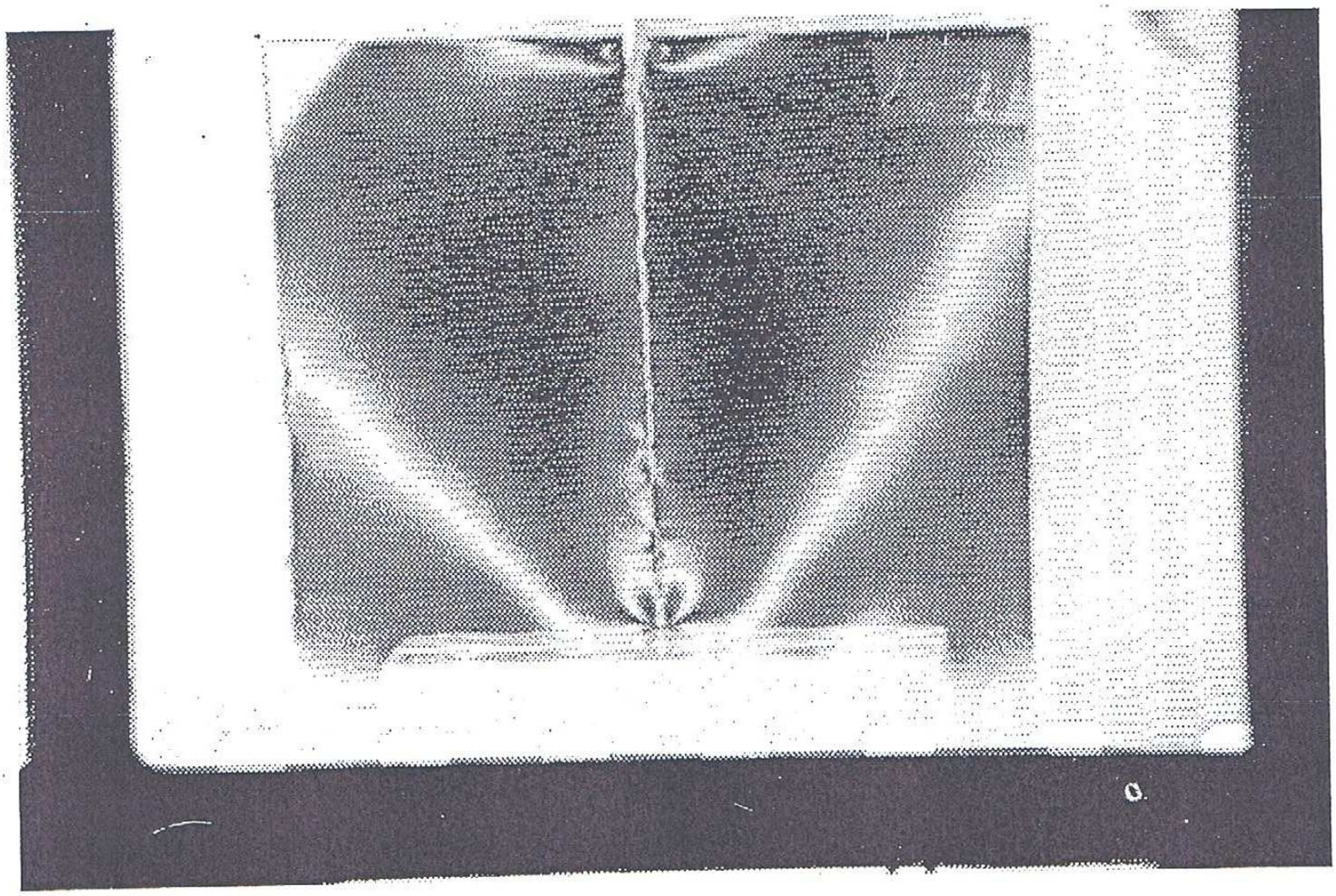
'p1-2h'			
Printed on:	Tue 19-Oct-1993 9:18		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0
Cells per inch:	120	Dot gain:	0%



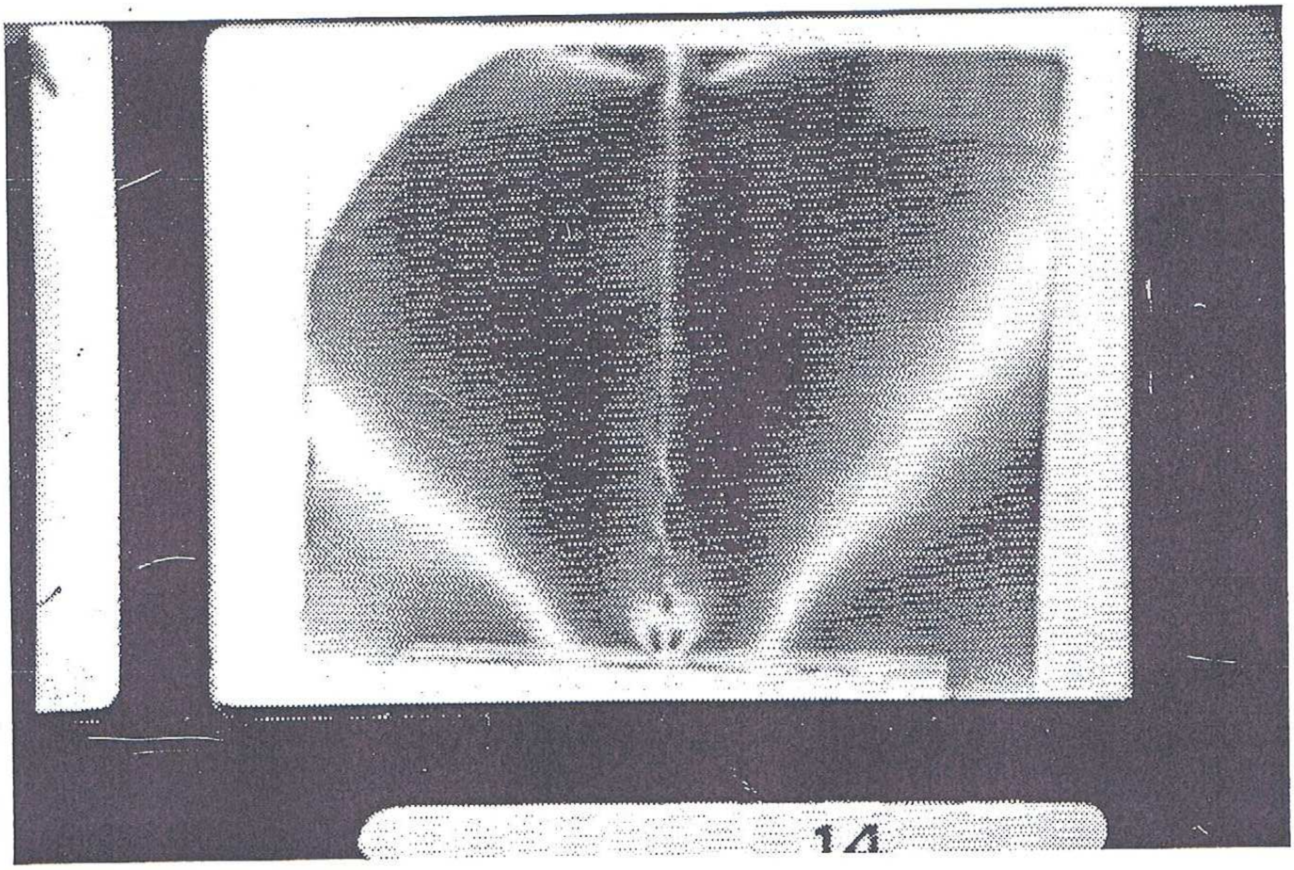
'p2-2h'			
Printed on:	Tue 19-Oct-1993 9:19		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0
Cells per inch:	120	Dot gain:	0%



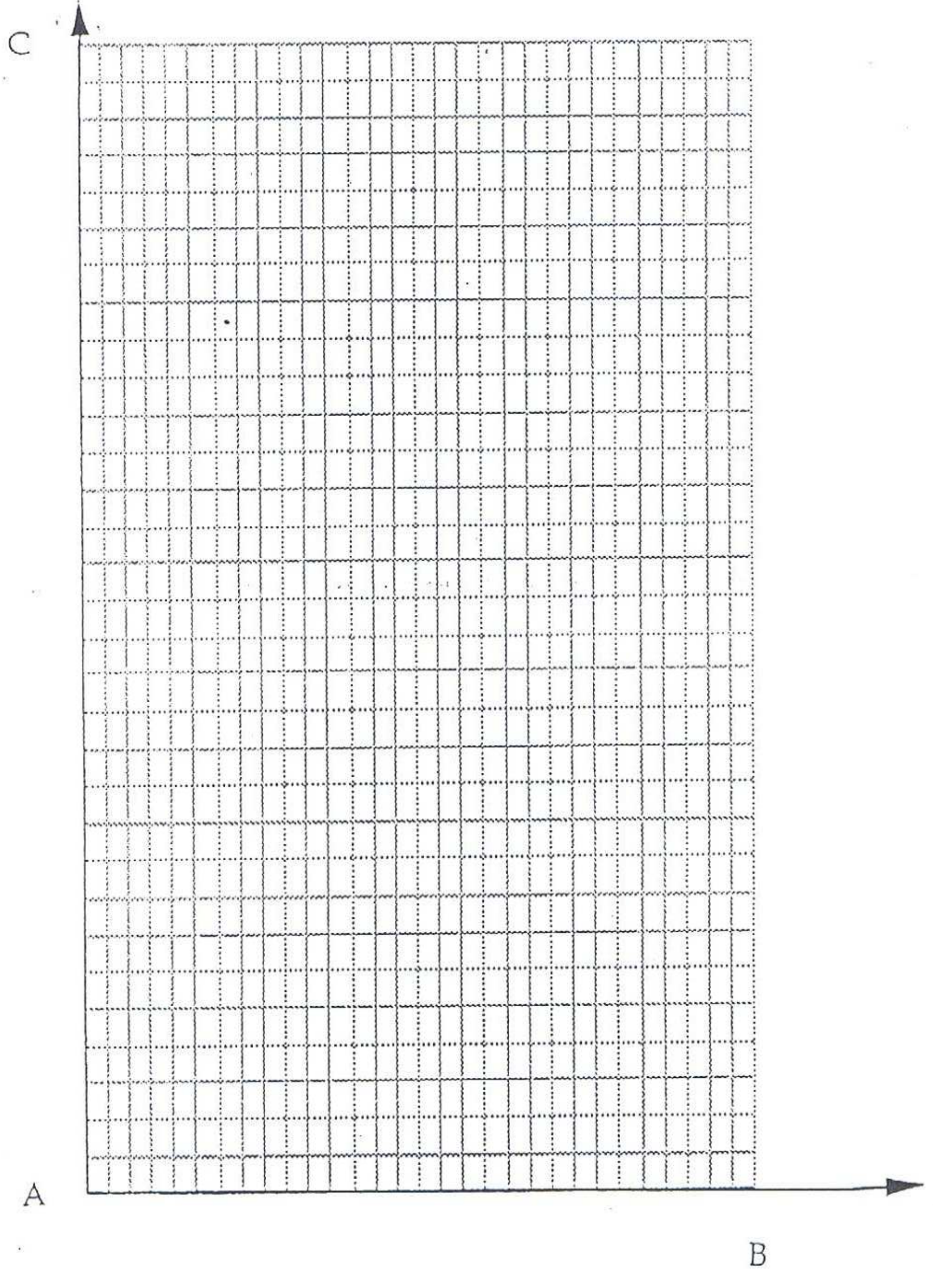
'p3-2h'			
Printed on:	Tue 19-Oct-1993 9:20		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0
Cells per inch:	120	Dot gain:	0%

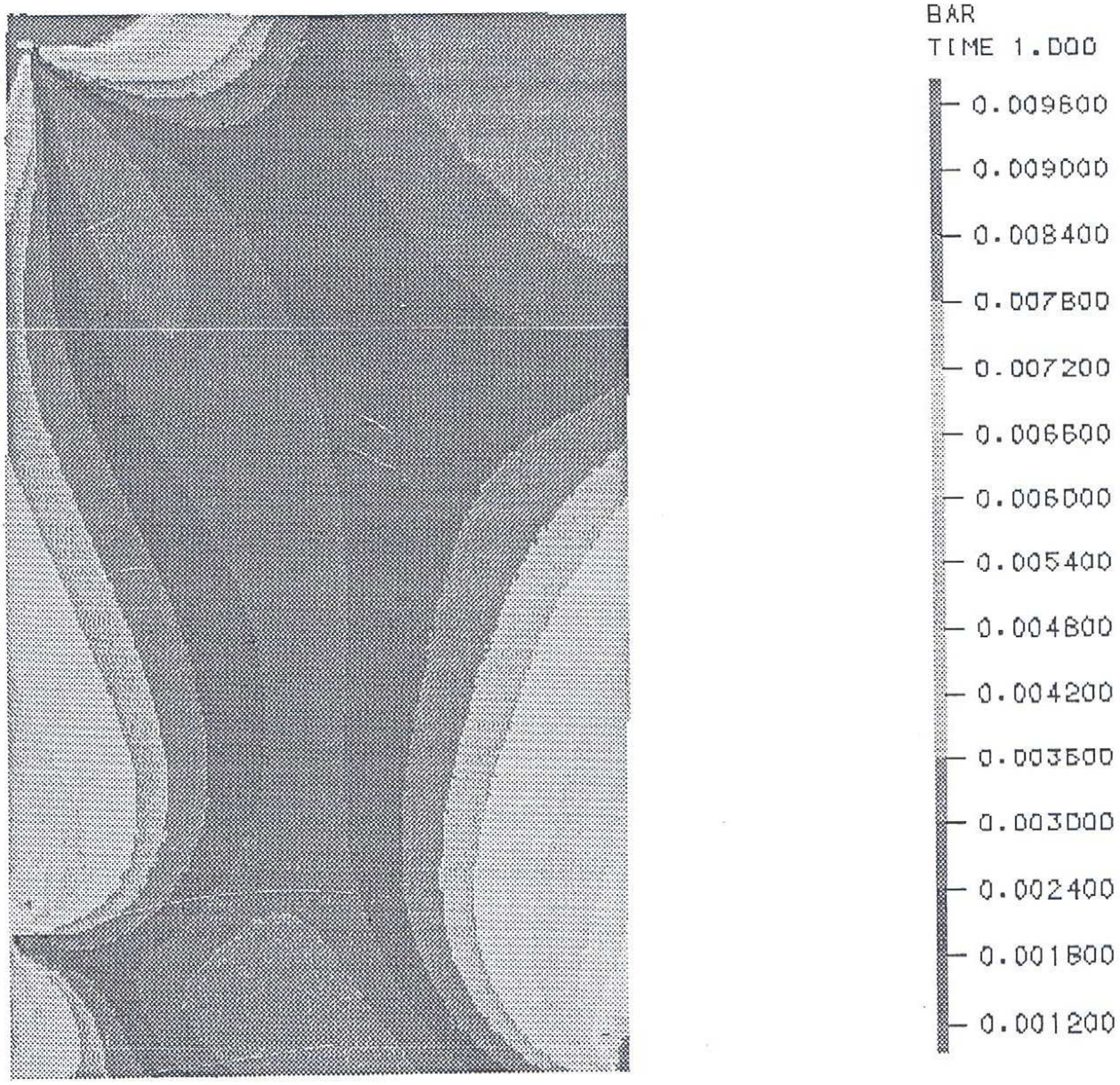


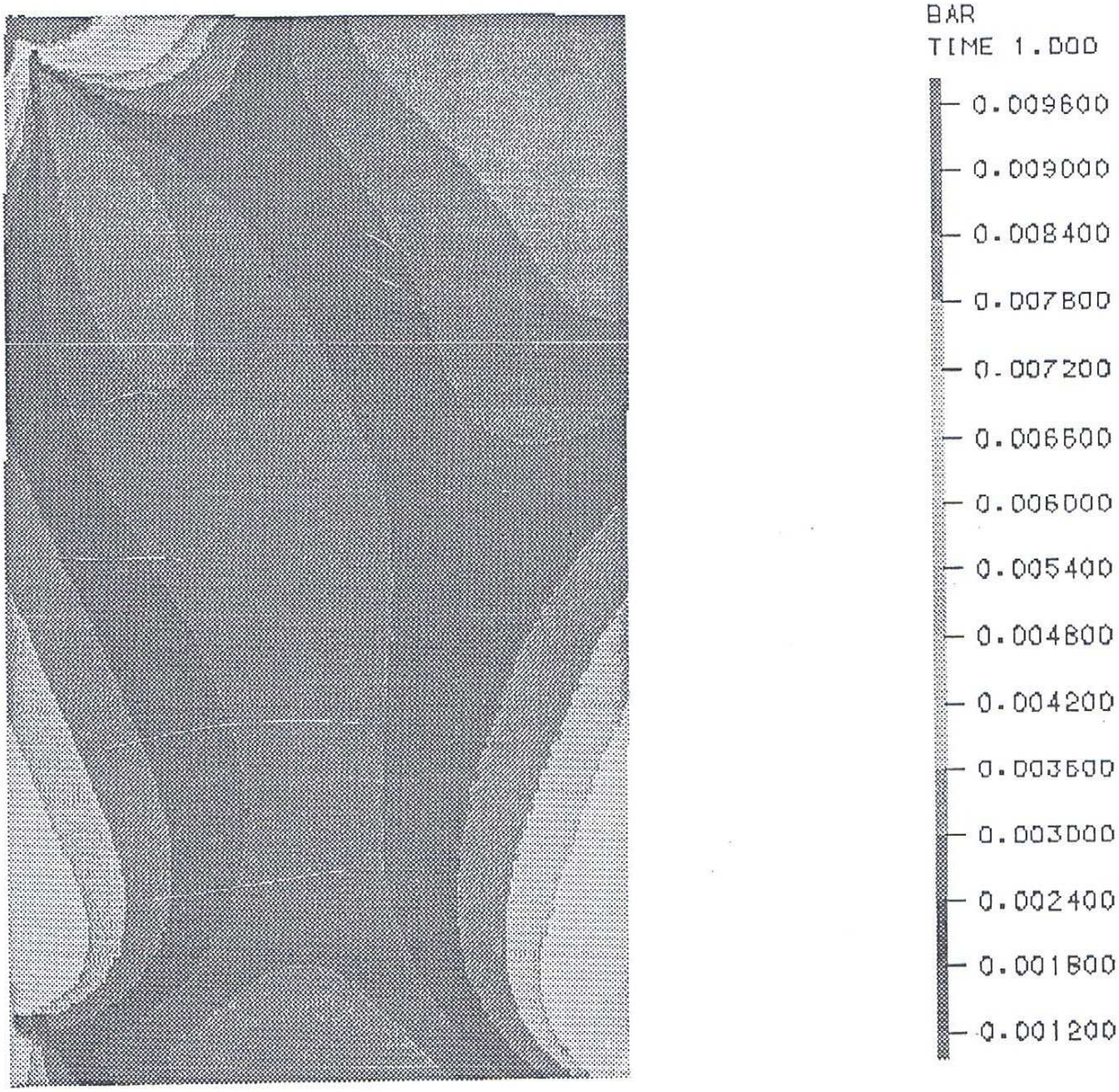
'p4-2h'			
Printed on:	Tue 19-Oct-1993 9:22		
Screen density:	Default	Screen angle:	Default
Dot pattern:	Default	Rotation:	0
Cells per inch:	120	Dot gain:	0%

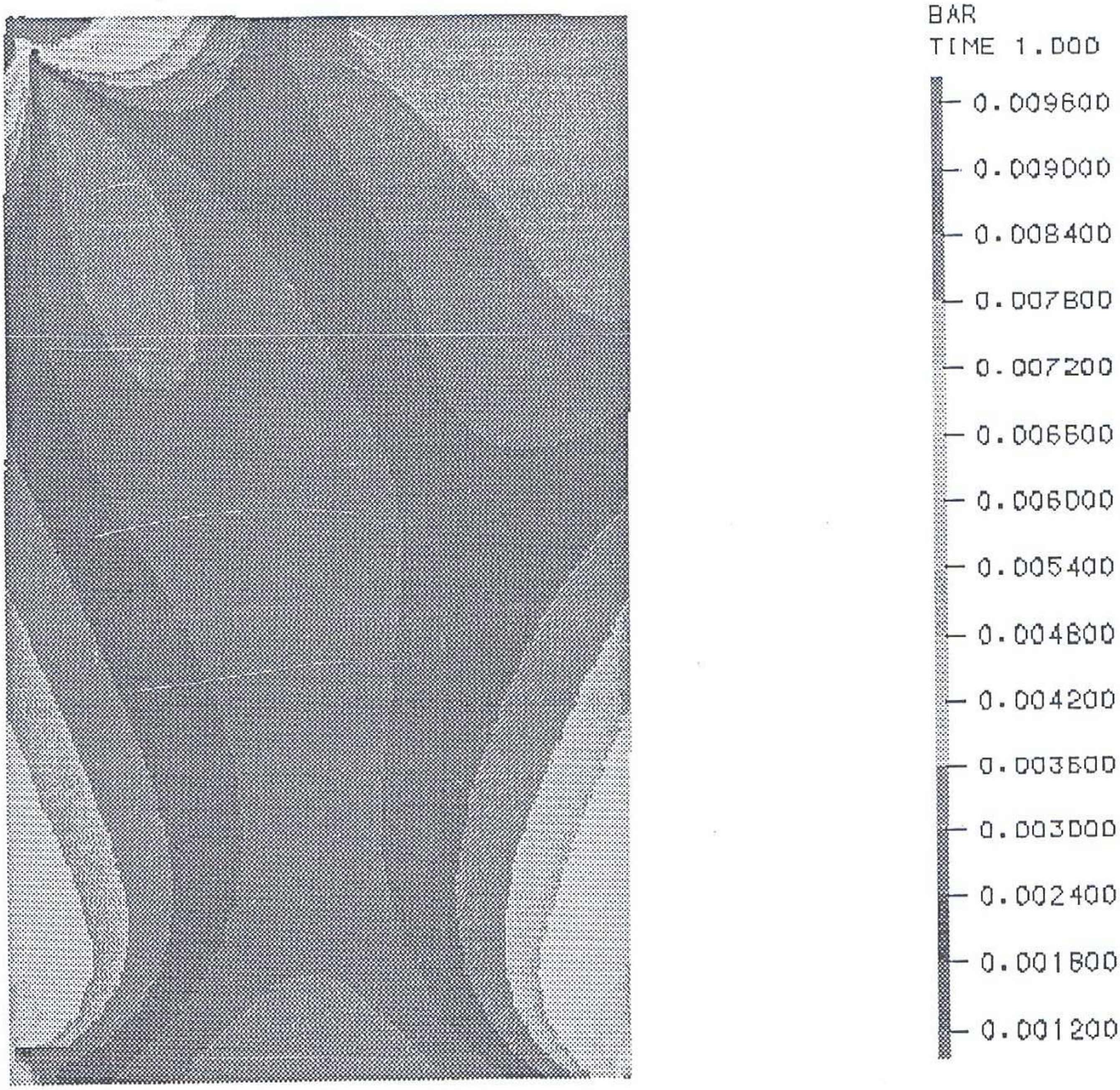


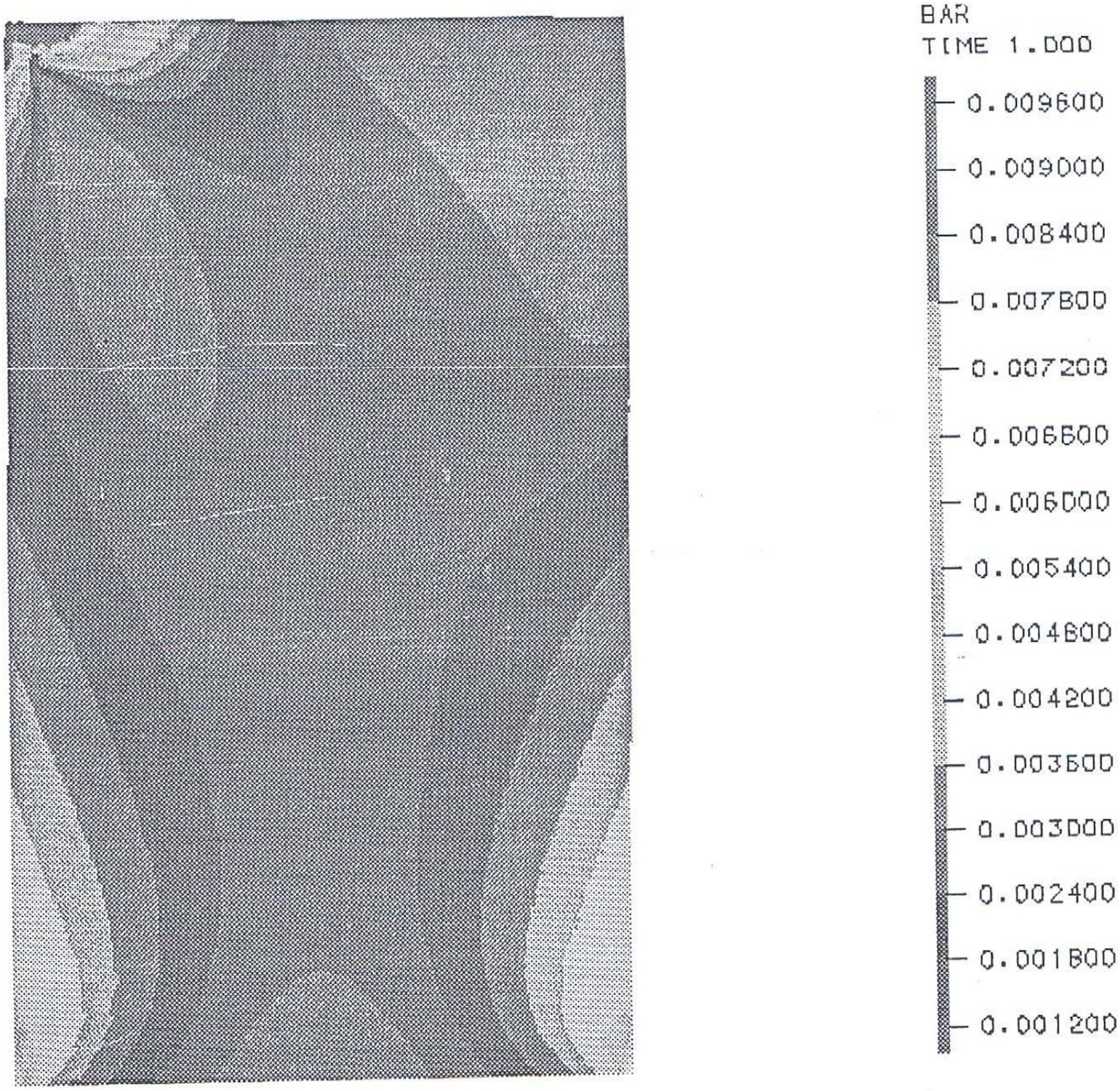
ORIGINAL XVMIN 0.000
XVMAX 9.000
0.9774 YVMIN 0.000
YVMAX 15.50

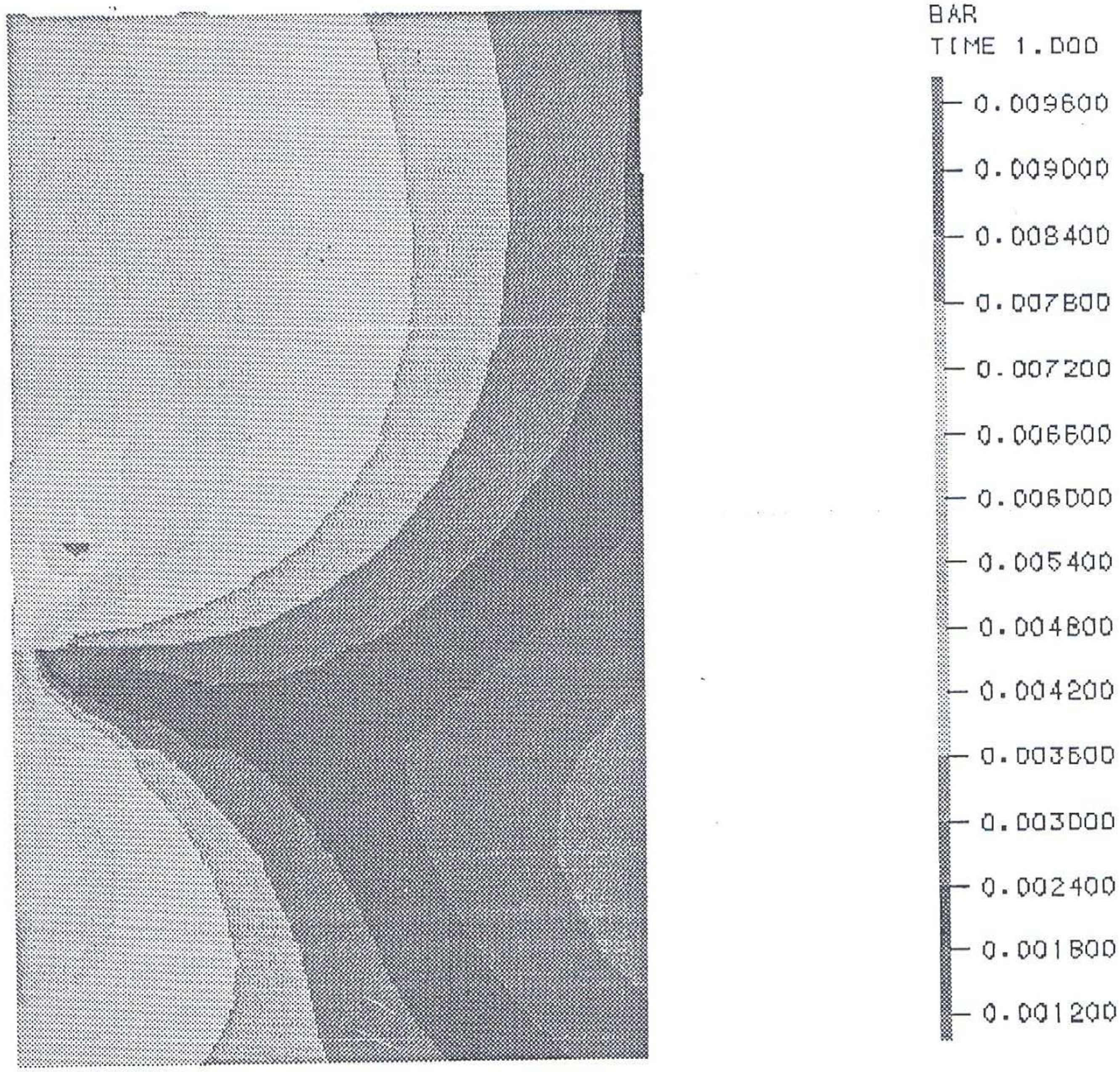


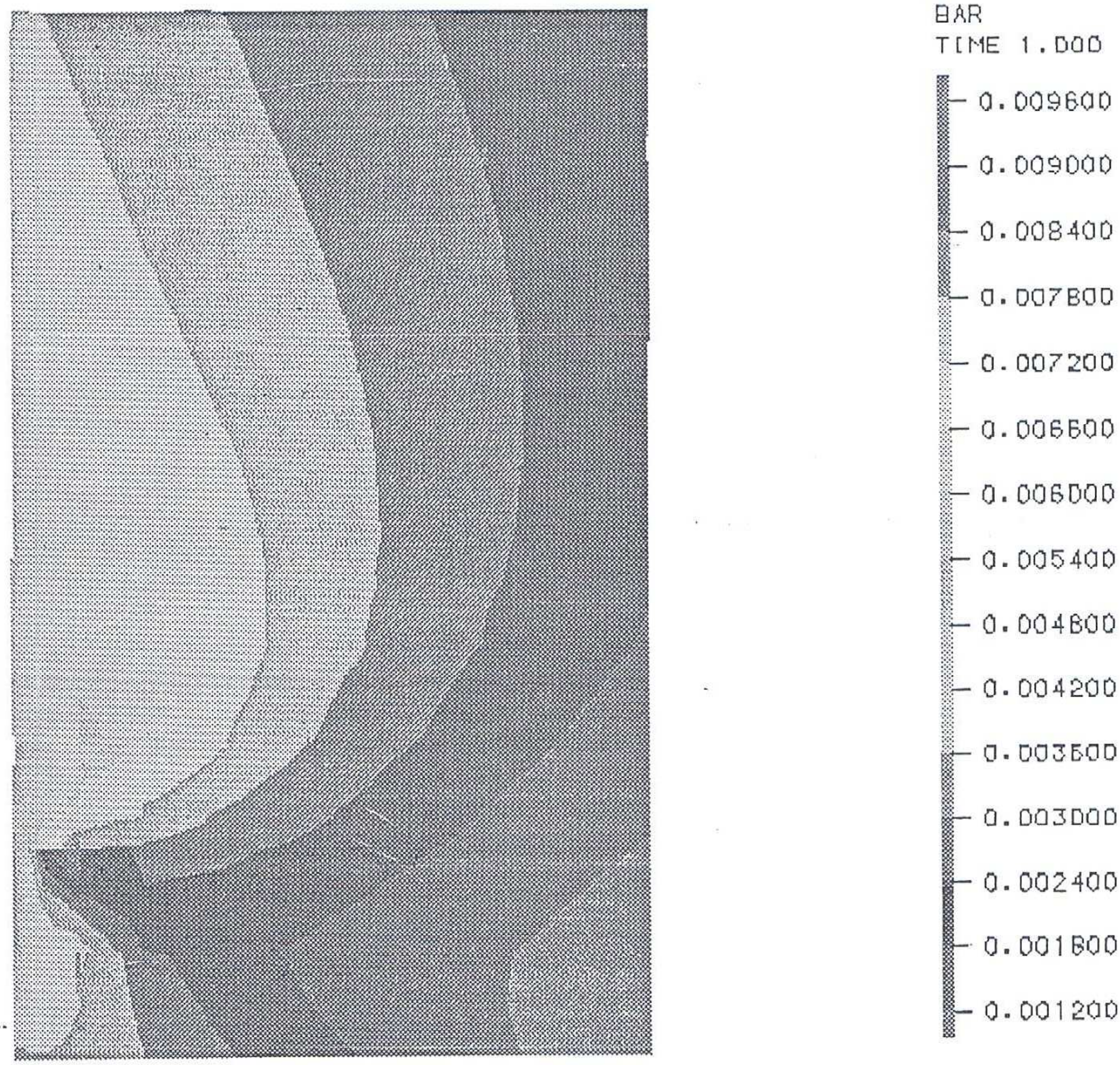

BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200


BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200

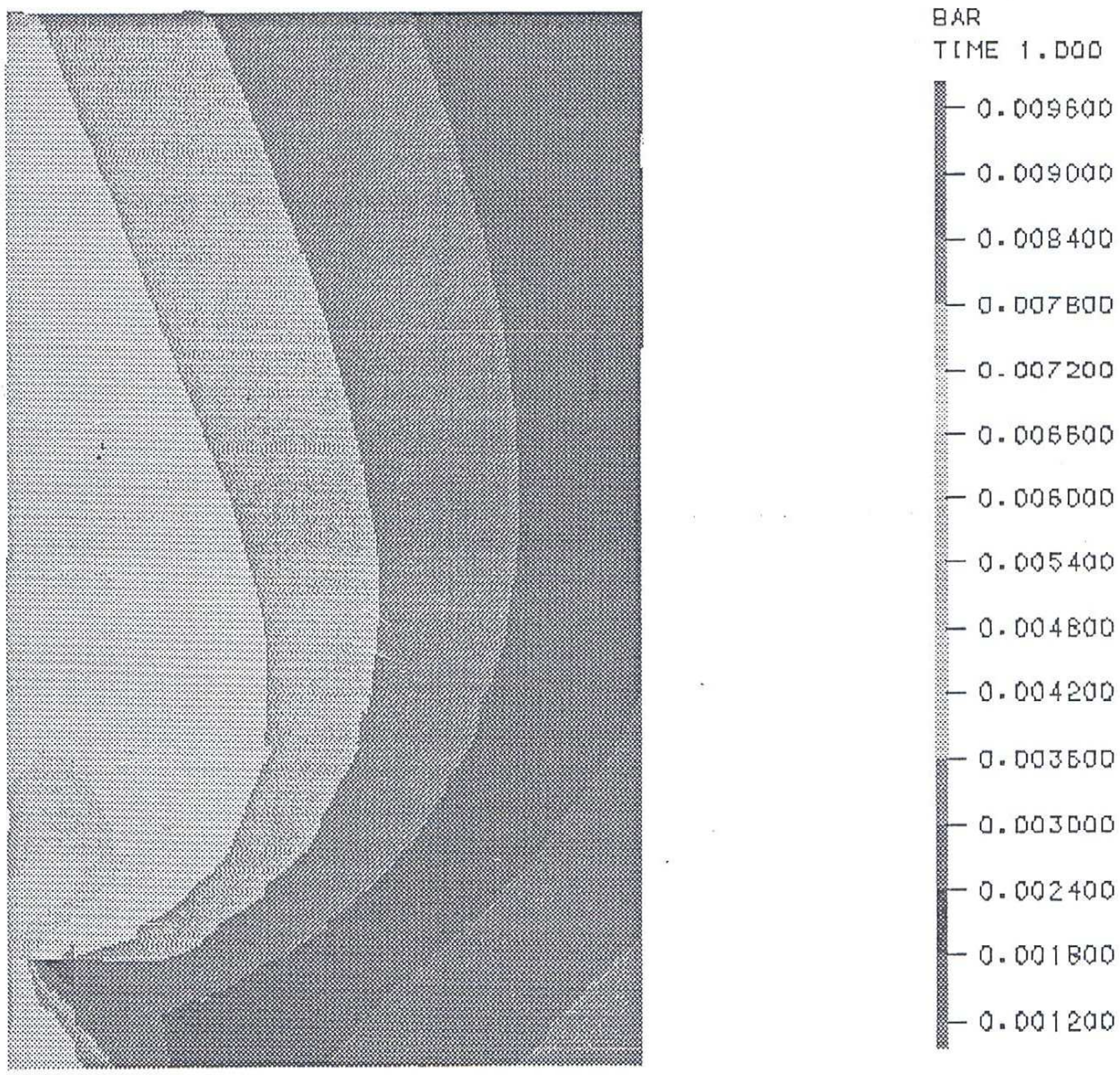

BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200

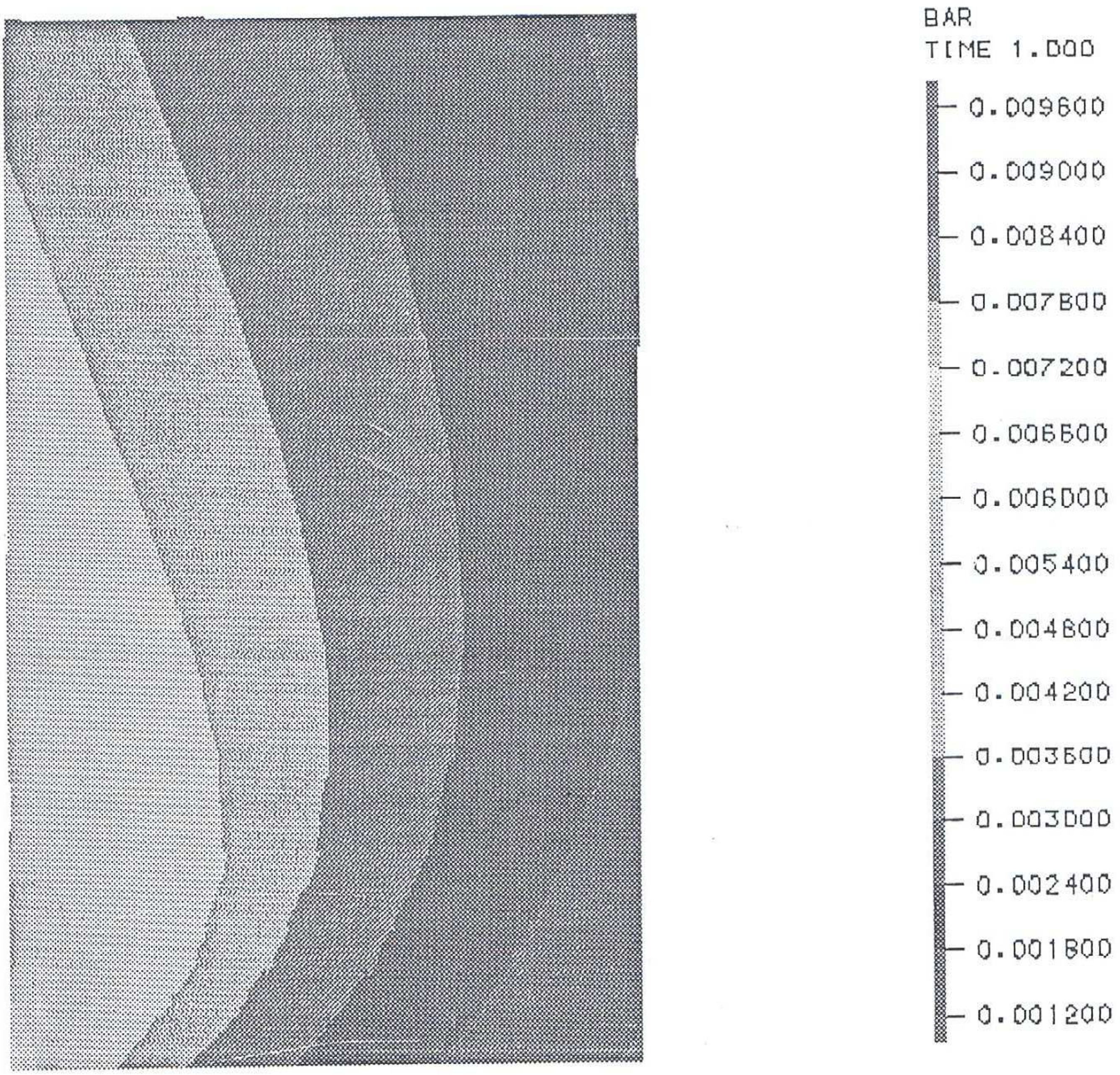

BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200


BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200



BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200


BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200


BAR
TIME 1.000
0.009600
0.009000
0.008400
0.007800
0.007200
0.006600
0.006000
0.005400
0.004800
0.004200
0.003600
0.003000
0.002400
0.001800
0.001200

Conclusions

The behaviour of cracks crossing interfaces is discussed. The system analysed consists of two dissimilar layers only. The problem of a crack growing towards an interface between two dissimilar materials is fundamental in the understanding of the behaviour of non-homogenous materials.

A primary investigation on a bimaterial system have been performed by using both experimental, numerical as well as analytic techniques to estimate the variation of the plane problem of crack that terminates at the interface of a bimaterial composite when loaded on its faces in a mode I way.

Modelling crack-tip fields, crack initiation and crack growth in bimaterial interfaces is essential for understanding failure processes in advanced materials.

An interface crack investigation is primarily important for the strength of the composite assessment because interfacial and intergranular fracture is common for such materials and usually defines the material overall strength qualities.

The problem about plate consists from two different materials with interface crack under bending load have a very practical interest. In particular case, when one of the materials is absolutely rigid, this problem comes to the problem of homogeneous plate with crack along the fixed edge under bending load till example.

Exact analytical solution of these problems is very difficult even for plate of simply form. Therefore numerical methods are expedient in these problems. But parameters of Stress-strain State have oscillating singularity at the crack tips.

A fringe patterns obtains from the experimental tests at the vicinity of the crack-tip for the different crack lengths at the same load level is essential for understanding failure processes in advanced materials.

The difference between the maximum and the minimum principle stresses obtained from the numerical investigation agreed well with that measured experimentally.